List of contributions

Keynote talks

Modularity for $\mathrm{GL}(2)$

Jack Thorne

Abstract not given. **Friday**

(Algebraic) aspects of modular forms (and quadratic forms)

Lynne Walling

Abstract not given. **Thursday**

The mysteries of L-values

Sarah Zerbes

L-functions are one of the central objects of study in number theory. There are many beautiful theorems and many more open conjectures linking their values to all kinds of arithmetic problems. I will talk about the mysteries surrounding these L-values and describe some of the progress that has recently been made towards understanding them. **Thursday**

Junior talks

Overconvergent modular forms via perfectoid methods

Chris Birkbeck

Classically, modular forms are defined as holomorphic functions on the upper half plane satisfying certain transformation properties or alternatively as sections of automorphic line bundles on modular curves. Now, if one is interested in studying p-adic properties of modular forms, then one can "p-adically interpolate" these line bundles to get spaces of overconvergent modular forms, which make precise the idea of modular forms living in p-adic families.

One might ask if there is a p-adic analogue of the description of modular forms as functions on the upper half plane. In this talk I will explain how one can use Scholze's infinite level modular varieties to give such a definition of overconvergent elliptic (and Hilbert) modular forms and some of its advantages. This is all work in progress joint with Ben Heuer and Chris Williams. **Thursday**

Tame torsion of Jacobians

Matthew Bisatt

Fix a positive integer g and rational prime p. Does there exist a genus g curve, defined over the rationals, such that the p-torsion field of its Jacobian is tamely ramified at every prime?

We give a constructive answer to this using the theory of endomorphisms. **Friday**

Low degree points on modular curves

Josha Box

Abstract not given. **Friday**

On the equivariant L-function at $s = 0$

Dominik Bullach

L-functions are a central object of study in modern number theory and there are far-reaching conjectures concerning special values of those. By way of an example, I will explain what we mean by equivariant versions of these conjectures and why they are interesting. **Wednesday**

Rapid Calculation of Multiple Modular Form Spaces

Kieran Child

Modular forms are fascinating structures which give rise to a number of important results in Number Theory, including relations on divisor functions, formulae for the sum of squares functions, and perhaps most famously Wiles' proof of Fermat's Last Theorem. Being able to compute the power series coefficients of these forms is very important, and this is most often performed one space at a time using the modular symbols method of Cremona. In this talk, I will present an alternative method using structural theory and the Eichler-Selberg trace formula. This method offers a considerable time saving over the modular symbols approach if one wishes to compute many modular form spaces at once. This talk is based on joint work with Min Lee. **Wednesday**

Wild Galois Representations

Nirvana Coppola

An important invariant associated to an elliptic curve is its l-adic Galois representation. In this talk I will consider elliptic curves over a local field with potentially good reduction and describe how to determine the Galois representation under the assumption that the image of inertia is non-abelian. **Wednesday**

The secret life of Fourier coefficients of Poincaré Series

Tiago Jardim da Fonseca

Poincaré series are among the first examples of holomorphic and weakly holomorphic modular forms. They are useful in many analytical questions, but their Fourier coefficients seem hard to grasp algebraically. In this talk, I will discuss the arithmetic nature of Fourier coefficients of Poincaré series, and explain that they are in fact cohomological invariants (periods). **Friday**

On Coleman's Conjecture for Circular Distributions

Alexandre Daoud

In 1989 Robert Coleman formulated a remarkable conjecture concerning so-called 'circular distributions' which effectively amounts to a precise prediction about the Galois module structure of the set of Euler systems defined over abelian extensions of $\mathbb{Q}$. In this talk I shall define circular distributions and state Coleman's conjecture. I will also discuss some recent work-in-progress towards a natural p-adic version of this conjecture.
Friday

Period Polynomials of Holomorphic and Non-Holomorphic Modular Forms

Joshua Drewitt

Abstract not given. **Thursday**

Galois module structure in wild extensions of $\mathbb{Q}$

Fabio Ferri

Abstract not given. **Wednesday**

(Real Quadratic) Arthurian Tales

Dan Fretwell

In recent years there has been a lot of interest in explicitly identifying the global Arthur parameters attached to certain automorphic forms. In particular Chenevier and Lannes were able to completely identify the full lists of Arthur parameters in the case of level 1, trivial weight automorphic forms for definite orthogonal groups of ranks 8, 16 and 24 (not a simple task!).

One finds interesting modular forms hidden in these parameters (e.g. Delta and a handful of special Siegel modular forms of genus 2). Comparing Arthur parameters mod p proves/reproves various Eisenstein congruences for these special modular forms, e.g. the famous 691 congruence of Ramanujan and, more importantly, an example of a genus 2 Eisenstein congruence predicted by Harder (which, up to then, had not been proved for even a single modular form!).

In this talk I will discuss recent work with Neil Dummigan on extending the above to definite orthogonal groups over certain real quadratic fields and try to tell the analogous Arthurian tales (mysteries included). **Friday**

Restrictions on endomorphism algebras of Jacobians

Pip Goodman

Zarhin has extensively studied restrictions placed on the endomorphism algebras of Jacobians of hyperelliptic curves $C : y^2 = f(x)$ when the Galois group $\mathrm{Gal}(f)$ is insoluble and 'large' relative to g the genus of C. But what can be said when $\mathrm{Gal}(f)$ is not 'large' or insoluble? We will see that for many values of g, much can be said if $\mathrm{Gal}(f)$ merely contains an element of 'large' prime order. **Thursday**

Anticyclotomic Euler systems for unitary groups

Andrew Graham

The collection of Heegner points on an elliptic curve E form an anticyclotomic Euler system and (when non-zero) provide examples of infinite-order rational points. In addition to this, the formula of Gross and Zagier describes a relationship between the derivative of the L-function of E and the height of such a point, thus providing instances of the Bloch-Kato conjecture in the analytic rank one case. I will describe a possible generalisation of this Euler system to conjugate self-dual representations of GL_{2n}. This is joint work with W. Shah. **Thursday**

Explicit descent by 3-isogeny on elliptic curves

Steven Groen

Descent by a rational isogeny has shown to be a useful tool in computing the rank of elliptic curves. After outlining the general theory, we recall the well-known theory of descent by 2-isogeny. A relatively new approach is descent by a 3-isogeny. In this thesis, we formulate an algorithm that computes the Selmer group of any rational 3-isogeny. We apply these techniques to a family of elliptic curves that allow both types of descent. We force elements into the Selmer group of the 3-isogeny, while we show by 2-descent that the elliptic curves have rank zero. This yields a construction of elements of order 3 in a Tate-Shafarevich group. **Wednesday**

The p-part of BSD for residually reducible elliptic curves of rank one

Giada Grossi

Let E be an elliptic curve over the rationals and p a prime such that E admits a rational p-isogeny satisfying some assumptions. In a joint work with J. Lee and C. Skinner, we prove the anticyclotomic Iwasawa main conjecture for E/K for some suitable quadratic imaginary field K. I will explain our strategy and how this, combined with complex and p-adic Gross-Zagier formulae allows us to prove that if E has rank one, then the p-part of the Birch and Swinnerton-Dyer formula for E/Q holds true. **Friday**

Formulas for the Gross-Stark units

Matthew Honnor

In the 1980's Tate stated the Brumer-Stark conjecture which, for a totally real field F with prime ideal $\mathfrak{p}$, conjectures the existence of a $\mathfrak{p}$-unit called the Gross-Stark unit. This unit has $\mathfrak{P}$ order equal to the value of a partial zeta function at 0, for a prime $\mathfrak{P}$ above $\mathfrak{p}$. In 2008 and 2018 Dasgupta and Dasgupta-Spieß, conjectured formulas for this unit. During this talk I shall explain Tate's conjecture and then the ideas for the constructions of these formulas. Time permitting, I will end by comparing the two formulas. **Wednesday**

Paths Of Character Sums

Ayesha Hussain

There is a strong history of work on partial sums of Dirichlet characters $\sum_{n\leq x}\chi(n)$, where χ is a non-principal character modulo odd prime q. In 1919, Pólya and Vinogradov proved that this sum is bounded by $\sqrt{q}\log q$. This bound has since been improved and the distribution of the maximal magnitude found. This talk will explore the polygonal path joining partial sums of Dirichlet characters modulo odd prime q. However, the limiting distribution of this path still remains a difficult and open question. We will discuss work to this end, incorporating ideas based on Kloosterman paths. The work is joint with J. Bober. **Friday**

Polyharmonic Maass forms of higher level

Gene Kopp

Polyharmonic Maass forms of weight k and depth d are functions on the upper half plane satisfying a modularity condition and vanishing under the d-th power of the weight k Laplacian. I will construct bases for spaces of level N polyharmonic Maass forms, generalising results of Lagarias and Rhoades in level 1. I will also give a formula for the leading coefficient of a rank 2 Hecke L-function of a real quadratic field at $s = 0$ (predicted by the Stark conjectures to be a 2-dimensional regulator) in terms of cycle integrals of polyharmonic Maass forms of depth 2. All results are joint work with Olivia Beckwith. **Wednesday**

Selmer groups of genus 3 curves in families

Jef Laga

The study of the arithmetic of curves in families is an active research area, which got a massive boost when Bhargava-Shankar determined unconditionally the average size of the n-Selmer group of the family of all elliptic curves over $\mathbb{Q}$, where n is less than six. Similar statements have been obtained for Selmer groups of hyperelliptic curves by Bhargava-Gross. These results have many diophantine consequences ('most' odd hyperelliptic curves have only one rational point), and it is fair to say that they are the main reason that Bhargava got his Fields medal. In this talk I will report on work in progress which extends this to certain families of non-hyperelliptic genus 3 curves, using a connection between singularity theory and Lie algebras. **Wednesday**

Plectic Galois actions on Hilbert modular varieties

Marius Leonhardt

Abstract not given. **Wednesday**

Quasi periods of modular forms

Ma Luo

Abstract not given. **Thursday**

Explicit methods for the Hasse norm principle and applications

André Macedo

Given an extension L/K of number fields, we say that the Hasse norm principle (HNP) holds if every non-zero element of K which is a norm everywhere locally is in fact a global norm from L. If L/K is cyclic, the original Hasse norm theorem states that the HNP holds. More generally, there is a cohomological description (due to Tate) of the obstruction to the HNP for Galois extensions.

In this talk, I will present recent work (joint with Rachel Newton) developing explicit methods to study this principle for non-Galois extensions. As a key application, I will describe how these methods can be used to characterize the HNP for extensions whose normal closure has Galois group A_n or S_n. I will also discuss the geometric interpretation of this principle and how it relates to the weak approximation property for norm one tori. **Friday**

A lower bound for the variance of generalized divisor functions in arithmetic progressions

Daniele Mastrostefano

The aim of this talk is to explain how we can find a lower bound for the variance in arithmetic progressions of a large class of multiplicative functions, referred to as generalized divisor functions. A direct corollary is a lower bound for the corresponding variance of any α-fold divisor function, for any complex number $\alpha \notin \{1, 0\} \cup -\mathbb{N}$, even when considering a sequence of parameters α converging in a proper way to 1. Our work builds on that of Harper and Soundararajan, who set the basics for the study of lower bounds for variances of complex sequences in arithmetic progressions and also handled the particular case of k-fold divisor functions $d_k(n)$, with $k \in \mathbb{N}_{\geq 2}$. **Friday**

Motivic periods

Nils Matthes

Abstract not given. **Thursday**

Minimal differentials of hyperelliptic curves

Simone Muselli

Let $C : y^2 = f(x)$ be a hyperelliptic curve over $\mathbb{Q}$, and let J be its Jacobian over $\mathbb{Z}$. One of the invariants in the statement of the Birch & Swinnerton-Dyer conjecture for J is its period. Computing (the local p-part of) this quantity relies on finding a basis for the global sections of (the determinant of) the dualizing sheaf of a regular model of $C_{\mathbb{Z}_p}$, for every prime number p. In this talk I will present an explicit formula to compute such a basis under certain conditions on the cluster picture of f. **Thursday**

Models of Hyperelliptic Curves Arising from Cluster Pictures

Sarah Nowell

For a hyperelliptic curve $C : y^2 = f(x)$, a huge amount of information can be extracted from the p-adic distances between the roots of $f(x)$. In particular, in collaboration with O. Faraggi, we have shown that this information is nearly all that is required to determine a model of any hyperelliptic curve. I will introduce a combinatorial way of visualising these p-adic distances through cluster pictures, and illustrate the motivation behind this theorem with some examples. **Wednesday**

On generalised Lebesgue-Ramanujan-Nagell equations

Vandita Patel

We describe a computationally efficient approach to resolving equations of the form $C_1x^2 + C_2 = y^n$ in coprime integers, for fixed values of C_1, C_2 subject to further conditions. We make use of a factorisation argument and the Primitive Divisor Theorem due to Bilu, Hanrot and Voutier. **Wednesday**

Statistics of Selmer Groups as Galois Modules

Ross Paterson

If we let $E/\mathbb{Q}$ run through a natural family of elliptic curves, then the average size of the 2-Selmer group of $E/\mathbb{Q}$ has been well studied, thanks to work of Heath-Brown, Swinnerton-Dyer, Kane, Poonen–Rains, Bhargava–Shankar and many others. If we also fix a quadratic number field $K/\mathbb{Q}$ and look at the 2-Selmer group of E/K as $E/\mathbb{Q}$ varies as before, then size is no longer the only interesting question. In fact, the 2-Selmer group of E/K is more than just an $\mathbb{F}_2$-vector space, it is an $\mathbb{F}_2$-representation of the Galois group of $K/\mathbb{Q}$. One might then ask the finer statistical question: What is the structure of this representation on average?

I will report on an ongoing project, the goal of which is to understand the statistical behaviour of these representations as $E/\mathbb{Q}$ varies in natural families. **Wednesday**

Non-ordinary Iwasawa theory for modular forms

Rob Rockwood

Abstract not given. **Friday**

Relative motives over Shimura varieties

Alex Torzewski

Given a Shimura variety S defined by a group G (e.g. a modular curve, in which case $G = \mathrm{GL}_2$), there is, almost by definition, a "canonical construction" which assigns to a representation of G a variation of Hodge structures on the Shimura variety. Similarly, there is also a construction which gives an l-adic sheaf for each representation. It is conjectured that these should both be motivic in origin, i.e. for each representation of G, there should be a way to construct a motive over S whose cohomology coincides with that given by the previous constructions. We motivate this with examples and present some new and existing partial results. **Thursday**

A geometric Jacquet-Langlands correspondence for paramodular Siegel threefolds

Pol van Hoften

It is an old idea of Serre that the classical Jacquet-Langlands correspondence between modular forms and quaternion modular forms can be realised geometrically. In this talk I will discuss an extension of these ideas to Siegel modular forms of genus two and paramodular level. We use this to prove the weight-monodromy conjecture for the Siegel threefold of paramodular level. Moreover we construct a geometric Jacquet-Langlands correspondence between GSp_4 and a 'definite' inner form, proving a conjecture of Ibukiyama. **Thursday**

Two formulations of Serre's conjecture

Hanneke Wiersema

Serre's conjecture, now a theorem of Khare-Wintenberger, states that all continuous, odd, irreducible mod p two-dimensional Galois representations arise from a modular form. The strong form specifies, amongst other things, the minimal weight of the modular form. Serre's notion of modularity has been reformulated which has led to a different notion of weights, called Serre weights. In a more general setting, Buzzard, Diamond and Jarvis define a set of Serre weights for which a given representation is expected to be modular. We show these versions are compatible when it comes to the minimal weight in the classical case and discuss generalisations of this. **Wednesday**

Congruences between modular forms of integer and half-integer weight

Sadiah Zahoor

In 1982, Jerrold Tunnell gave an elegant characterisation of congruent numbers. He linked congruent numbers to Fourier coefficients of half-integral weight modular forms. We are intrigued to examine them by looking into congruences of Fourier coefficients of half-integer weight forms with Fourier coefficients of classical modular forms. We shall start with a well known congruence between integer weight new forms and try to lift it to a congruence between half-integer weight Kohnen New-forms. Beginning with one dimensional space of New-forms, we eventually increase the dimension and see what happens. It seems that the structure of Hecke Algebra for half-integer modular forms makes it challenging to do so. **Wednesday**