YRANT III

18^{th} - 20^{th} August, 2021

The (virtual) University of Bristol

Organisers:
Nirvana Coppola, Dan Fretwell, Pip Goodman, Ayesha Hussain, Simone Muselli, Andrei Seymour-Howell

Introduction

It is our great pleasure to welcome you to YRANT III, the conference that COVID tried to stop! After the huge success that was YRANT II, held at Warwick in 2019, we were feeling nothing but pure enthusiasm and eagerness when we offered to host the next event at Bristol in 2020. We knew nothing of the perils that were about to unfold.

Of course, we all know the story, and after having to postpone a few times we eventually decided to move the conference online. However, in doing this we have been able to attract many participants from all over the world and schedule more talks than ever before. The interest has been overwhelming!

In this booklet you'll find the full schedule and list of abstracts. Each speaker has been assigned to one of three parallel sessions on Zoom (the link can be found in the welcome email).

Due to the unique nature of the conference this year we needed to find new ways for people to socialise and discuss (after all, this is partly what YRANT is all about). To this end we have created both a Gathertown space and a Slack channel (links found in the welcome email). We strongly encourage participants to use these, as they are the closest we could get to recreating the welcoming and inclusive atmosphere that YRANT always inspires. In particular, we have created three rooms in Gathertown that correspond to each of the three parallel sessions. We would very much like speakers to make use of these during the break session directly after their talk. This makes it easier to be located for questions/discussions.

Now for the future. How can we possibly talk about YRANT III without also mentioning YRANT IV? Tradition dictates that the organisers of the next event must volunteer by the end of this event. If this sounds like something you want to do then please get in touch with one of us, or fill in the sign up sheet in Gathertown.

Finally, we hope that you enjoy the conference. Any feedback, positive or negative, would be greatly appreciated (not just for us but for future organisers!). We would also like to take this opportunity to thank both the Heilbronn Institute and the LMS, for continued support of this conference.

Bristol, August 2021
Nirvana, Dan, Pip, Ayesha, Simone, Andrei

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	III	**Samuel Frengley**	N-Congruences Between Quadratic Twists of Elliptic Curves
10:30 - 11:00	I	**Jay Swar**	Selmer Spaces and (Derived) Symplectic Geometry
	II	**Pasupulati Sunil Kumar**	Note on the p-divisibility of class numbers of an infinite family of imaginary quadratic fields
	III	**Tarun Dalal**	The structure of Drinfeld modular forms of level $\Gamma_0(T)$ and applications
11:00 - 11:30	I	**Jef Laga**	Rational points and Selmer groups of genus 3 curves
	II	**Mar Curcó Iranzo**	Generalized modular jacobians
	III	**Francesco Battistoni**	Arithmetic equivalence for non-geometric extensions of global function fields
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16:00 - 17:00	Main	**Samir Siksek**	On the asymptotic Fermat conjecture

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	II	**Lorenzo Stefanello**	Local Galois module theory: An overview
	III	**Anwesh Ray**	Arithmetic statistics and the Iwasawa theory of elliptic curves
10:30 - 11:00	I	**Piotr Miska**	p-adic denseness of fractions of values of polynomials with integer coefficients
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	III	**Mohit Mishra**	Class group of real cyclotomic fields
11:00 - 11:30	I	**Josh Coyston**	Polynomials with Equivalent Mahler Measures
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Thursday 19th August - Afternoon

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	II	**Vinodkumar Ghale**	Congruent numbers, Heron triangles and elliptic curves
	III	**Steven Groen**	The p-torsion of Jacobians of Artin-Schreier curves
14:30 - 15:00	I	**Jaitra Chattopadhyay**	On Non-Pólya bi-quadratic fields with a Euclidean ideal class
	II	**Stevan Gajović**	Curves that meet the bound in the method of Chabauty and Coleman
	III	**Christopher Keyes**	Fields generated by points on superelliptic curves
15:00 - 15:30	I	**Yu Kuang**	On Galois-Gauss sums for weakly ramified characters
	II	**Nihar Gargava**	Lattice packings through division algebras
	III	**Lian Duan**	Principal version of the Chebotarev density theorem and class group
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	III	**Sam Streeter**	Campana points and powerful norms
10:30 - 11:00	I	**Philippe Michaud-Rodgers**	A Unique Perfect Power Decagonal Number
	II	**Ariel Weiss**	Elements of prime order in Tate-Shafarevich groups of abelian varieties over $\mathbb{Q}$
11:00 - 11:30	I	**Maleeha Khawaja**	The Fermat equation over real biquadratic fields
	II	**Dominik Bullach**	The equivariant Tamagawa Number Conjecture for abelian extensions of imaginary quadratic fields
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Friday 20th August - Afternoon

Time (BST)	Room	Speaker	Title (Click for abstract)
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	III	**Caleb Springer**	Groups of rational points of abelian varieties over finite fields
14:30 - 15:00	I	**Tung Nguyen**	Fekete Polynomials, Quadratic Residues, and Arithmetic
	II	**Jesse Pajwani**	The p-adic Section Conjecture and valuations
	III	**Pedro-José Cazorla García**	The kryptonite of a number theorist
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15:30 - 16:30	Main	**Tim Dokchitser**	Small Galois groups over $\mathbb{Q}$ and $\mathbb{Q}(t)$

Academic Speakers

Small Galois groups over $\mathbb{Q}$ and $\mathbb{Q}(t)$

20th August
3:30pm
Main Room

Tim Dokchitser
University of Bristol

The Inverse Galois Problem asks whether every finite group occurs as a Galois group over $\mathbb{Q}$. This question and its variant over $\mathbb{Q}(t)$ have driven a lot of research. There is a variety of methods of constructing Galois groups, and they draw techniques from algebraic geometry, p-adic numbers, the theory of finite groups, Kummer theory, braid groups, group cohomology, and elliptic and modular curves, to name a few. I would like to give a quick overview of where we stand and mention some new methods and results, including families for small groups over $\mathbb{Q}$ and $\mathbb{Q}(t)$.

Depth and the local Langlands correspondence

19th August
4:00pm
Main Room

Beth Romano
University of Oxford

The local Langlands correspondence is a kaleidoscope of conjectures that links local Galois theory, complex Lie theory, and the representation theory of p-adic groups. The focus of this talk will be examples that illuminate the interactions between these areas. In particular, we'll look at the notion of depth, which comes from considering certain filtrations in p-adic groups, and which is mirrored in ramification subgroups of local Galois groups. If time permits, I'll talk about new results in the depth-zero part of the correspondence.

On the asymptotic Fermat conjecture

18th August
4:00pm
Main Room

Samir Siksek
University of Warwick

The asymptotic Fermat conjecture states that for a number field K there is a constant B_K such that for primes $p \geq B_K$ the only K-rational points on the Fermat curve $X^p + Y^p + Z^p = 0$ lie on the lines $X = 0$, $Y = 0$, $Z = 0$, $X + Y + Z = 0$. In this talk we survey joint work with Nuno Freitas, Alain Kraus and Haluk Sengun, on the asymptotic Fermat conjecture, proving AFC for several infinite families of number fields.

Contributed talks

Arithmetic equivalence for non-geometric extensions of global function fields

18th August
11:00am
Room III

Francesco Battistoni
Université Bourgogne Franche-Comté

Two finite separable extensions of $\mathbb{F}_q(T)$ are said to be arithmetically equivalent if, up to finitely many exceptions, every prime ideal of $\mathbb{F}_q(T)$ decomposes with same inertia degrees in the two extensions. This property is known to be equivalent to a relation intercurring between the Galois groups of the extensions, known as Gassmann equivalence, and this group-theoretic approach has proved to be crucial in studying arithmetic equivalence. In joint work with Hassan Oukhaba, we review the theory of arithmetic equivalence for extension of global function fields, extending previous results to the case of non-geometric extensions, i.e. extensions with field of constants bigger than $\mathbb{F}_q$. We also present explicit examples of arithmetically equivalent and non-isomorphic extensions of $\mathbb{F}_2(T)$ which are not equivalent over $\mathbb{F}_4(T)$, solving a specific inverse Galois problem.

Chow dilogarithm and reciprocity laws

19th August
2:00pm
Room I

Vasily Bolbachan
Higher School of Economics, Math Department, Moscow

I want to explain the solution of A. Goncharov's conjecture about strong reciprocity laws. As an application for any algebraically closed field of characteristic zero and any integer $q \geq 2$, we get a surjective map from the cubical Higher Chow group $CH^q_{\square}(F, q+1) \otimes \mathbb{Q}$ to the $(q-1)$-th cohomology of polylogarithmic complex of weight q. Main idea is the construction of norm map on so-called strong reciprocity laws in the spirit of Bass, Tate, Suslin and Kato. Also this result gives new relations for Bloch-Wigner dilogarithm parametrized by arbitrary projective surfaces together with four rational functions.

19th August
10:00am
Room I

Special values of zeta functions of totally real number fields

Zakariae Bouazzaoui
Moulay Ismail University of Meknès

If p is an odd prime number and r is a positive integer larger than 2 whose p-adic valuation is s, we consider the question of existence of totally real number fields K of degree r such that the p-adic valuation of the value at $2-p$ of the zeta function of K is exactly $-1-s$.

20th August
11:00am
Room II

The equivariant Tamagawa Number Conjecture for abelian extensions of imaginary quadratic fields

Dominik Bullach
King's College London

The equivariant Tamagawa Number Conjecture is a far-reaching equivariant refinement of the analytic class number formula. I will motivate the conjecture and mention recent joint work with Martin Hofer that establishes new cases of it.

20th August
2:30pm
Room III

The kryptonite of a number theorist

Pedro-José Cazorla García
University of Manchester

With the proof of Fermat's Last Thorem, a whole new strategy was created in order to attack Diophantine equations, which is now called the modular method. While extremely useful to discard the existence of solutions, this approach presents a fatal weakness: if a solution does exist, it is guaranteed to fail. In this talk, we shall present some examples of this phenomenon, including some equations which are obstructed by the existence of solutions to a DIFFERENT equation.

19th August
2:30pm
Room I

On Non-Pólya bi-quadratic fields with a Euclidean ideal class

Jaitra Chattopadhyay
Indian Institute of Technology Guwahati

An algebraic number field K is called a Pólya field if the ring of integer-valued polynomials has a regular basis. This behavior is governed by the Pólya group $Po(K)$, a particular subgroup of the ideal class group Cl_K of K. The study of Pólya groups for various number fields is of considerable interest in recent times. In this talk, we shall compute the Pólya groups of three possibly infinite families of bi-quadratic fields, two of which are known to have non-principal Euclidean ideal class under mild conditions. This is a joint work with Anupam Saikia.

Linear equations on Drinfeld modules

18th August
3:00pm
Room III

Yen-Tsung Chen
National Tsing Hua University

Let K be a number field and E be an elliptic curve defined over K. Given finitely many K-rational points on E, Masser proved that there is an explicit upper bound for the size of the generators of linear relations among those points. This upper bound depends on the number of points, Neron-Tate height on $E(K)$, and the size of the torsion subgroup of $E(K)$. Let L be a finite extension of the rational function field over a finite field. We aim to study an analogue of Masser's result for finitely many L-rational points on a given Drinfeld module defined over L.

Selmer groups attached to mod p Bianchi modular forms

18th August
3:00pm
Room I

Lewis Combes
University of Sheffield

Informally, it is expected that special values of L-functions associated to Galois representations should control the orders of certain Selmer groups associated to those representations. The aim of this talk is to explore how to explicitly compute both sides of this conjectural relationship in the case of representations arising from mod p Bianchi modular forms, with an eye towards particularly those representations attached to torsion classes, per Scholze's 2015 paper proving their existence.

Polynomials with Equivalent Mahler Measures

19th August
11:00am
Room I

Josh Coyston
Royal Holloway, University of London

The Mahler measure is used to show how far away a polynomial's roots are from being in the unit circle. We explore under what conditions two integer polynomials can have the same Mahler measure. From here, we look at a conjecture which argues that Mahler measure values are, up to a set of conditions, unique to certain polynomials. We end with a brief look at my attempts to prove this conjecture, and what the future holds.

18th August
11:00am
Room II

Generalized modular jacobians

Mar Curcó Iranzo

Utrecht University

(Generalized) Jacobians are important abelian varieties for studying curves. On the other hand, modular curves are important objects for studying elliptic curves with special properties. In particular, rational torsion points on (generalized) modular jacobians are related to elliptic curves with (non-trivial) torsion. My reaserch involves studying the rational torsion part and other properties of (generalized) modular jacobians, for which many exciting conjectures exist.

18th August
10:30am
Room III

The structure of Drinfeld modular forms of level $\Gamma_0(T)$ and applications

Tarun Dalal

Indian Institute of Technology Hyderabad

In this talk, we describe the structure of the R-algebra of Drinfeld modular forms of level $\Gamma_0(T)$. Using this structure theorem, we study the properties of the weight filtration of Drinfeld modular forms of level $\Gamma_0(T)$. Finally, we prove a result on mod-$\mathfrak{p}$ congruences for Drinfeld modular forms of level $\Gamma_0(\mathfrak{p}T)$ for $\mathfrak{p} \neq (T)$.

18th August
2:30pm
Room I

Computing p-adic L-functions using Hilbert Eisenstein series

Håvard Damm-Johnsen

University of Oxford

In 1973, Serre defined p-adic modular forms as p-adic limits of modular forms, through which he gave a sleek definition of the Leopoldt-Kubota p-adic L-function by looking at the constant term of a limit of Eisenstein series. This was extended by Deligne-Ribet to totally real number fields using Hilbert modular forms, and forthcoming work by Lauder and Vonk gives an explicit algorithm for computing p-adic L-functions of totally real number fields using Serre's idea. We discuss the construction, extensions, and applications.

Deforming cyclic covers in towers

18th August
10:00am
Room II

Huy Dang
Vietnam Institute of Mathematics

There are many interesting phenomena in positive characteristics. For instance, there exist covers of curves whose numbers of branch points are different but still lie in the same flat family. In this talk, we briefly discuss the process of showing that a (flat) equal-characteristic $p > 0$ deformation of a cyclic sub-cover (of a curve) always extends to one for the whole cover. The crucial technique is the study of local covering's degeneration using the refined Swan conductors, which generalize the classical perfect residue case.

Principal version of the Chebotarev density theorem and class group

19th August
3:00pm
Room III

Lian Duan
Colorado State University

Given a Galois extension K of $\mathbb{Q}$, and take C to be a conjugate class of $Gal(K/\mathbb{Q})$. How probable is it that a prime integer p has its corresponding Frobenius class equal to C and p factors as product of principal ideals of K? In this talk, we provide an answer to this question in terms of the class number of K. Moreover, we will show that such density will in fact totally determine the class group of K. Using this, we also provide a way to check the non-splitting of the Hilbert exact sequence. This is a joint work with Kelly Emmrich, Ning Ma and Xiyuan Wang.

N-Congruences Between Quadratic Twists of Elliptic Curves

18th August
10:00am
Room III

Samuel Frengley
University of Cambridge

Elliptic curves are said to be N-congruent if their N-torsion subgroups are isomorphic as Galois modules. Halberstadt and Cremona-Freitas showed that the problem of finding p-congruences between an elliptic curve and a quadratic twist is equivalent to finding rational points on the modular curve associated to the normaliser of a Cartan subgroup of $GL_2(\mathbb{F}_p)$. We discuss an extension of this result to composite N and use this to construct explicit examples.

19th August
2:30pm
Room II

Curves that meet the bound in the method of Chabauty and Coleman

Stevan Gajović

University of Groningen

The method of Chabauty and Coleman is a powerful p-adic method to determine rational points on curves that satisfy a certain rank condition. It gives an upper bound for the number of rational points on such curves. However, it is quite unlikely that this bound is reached; there are only a few curves known to do so. It motivates the following question. For a fixed genus g, can we find a curve of genus g with the number of rational points attaining this upper bound? We construct examples conditionally on the rank and exhibit concrete examples of curves of genus up to five. We use Magma for our computations.

19th August
3:00pm
Room II

Lattice packings through division algebras

Nihar Gargava

École polytechnique fédérale de Lausanne

We will show the existence of lattice packings in a sparse family of dimensions. This construction will be a generalization of Venkatesh's lattice packing result of 2013. In our construction, we replace the appearance of the cyclotomic number field with a division algebra over the rationals. This improves the best known lattice packing bounds in many dimensions.

19th August
2:00pm
Room II

Congruent numbers, Heron triangles and elliptic curves

Vinodkumar Ghale

BITS-Pilani, Hyderabad Campus, India

The numbers which occur as an area of a rational triangle with some given angle θ are of great interest among the number theorists. In this talk, we shall briefly recall congruent numbers, Heron triangles and the correspondence between congruent numbers and the elliptic curves. We shall also talk about a generalization of this idea by Goins and Maddox for rational triangles. We shall conclude with a brief mention of speaker's collaborative work on the relationship between certain Heron triangles and the rank of elliptic curves associated with them.

Solving A Quartic Family of Relative Thue Equations

18th August
3:00pm
Room II

Eva Goedhart
Franklin & Marshall College

Similar to E. Thomas' work, we define a parametrized family of Thue equations over imaginary quadratic integers. After a brief introduction of the key terms, I will present some preliminary work on solving a quartic family of Thue equations using the hypergeometric method and Baker's method. This is joint work with Bernadette Faye, Benjamin Earp-Lynch, and Daniel Wisniewski.

The p-torsion of Jacobians of Artin-Schreier curves

19th August
2:00pm
Room III

Steven Groen
University of Warwick

The p-torsion of abelian varieties is easily described if the characteristic of the ground field is different from p. On the other hand, in characteristic p the multiplication by p map is inseparable and the p-torsion can take various forms. We specialise to the situation when the abelian variety in question is the Jacobian of an Artin-Schreier curve. Artin-Schreier curves are given by $Y : y^p - y = f(x)$ for some rational function f. It is known that in characteristic 2 the orders of the poles of f completely determine the isomorphism class of $\mathrm{Jac}(X)[p]$. We exhibit how this fails in odd characteristic.

Fields generated by points on superelliptic curves

19th August
2:30pm
Room III

Christopher Keyes
Emory University

We give an asymptotic lower bound for the number of degree n number fields, with discriminant bounded by X, which are the minimal field of definition for an algebraic point on a superelliptic curve C defined over the rational numbers. For C fixed and given by the affine equation $y^m = f(x)$ where f has degree d, the lower bound applies for all sufficiently large n divisible by $gcd(m, d)$ and takes the form X^c, where c is an explicit constant approaching $1/m^2$ as n grows without bound. This bound may be determined by explicit parameterization methods and accounting for the fact that many polynomials may give rise to isomorphic number fields. In the special case where C has a rational point, these methods are refined to give a similar lower bound for fields of all sufficiently large degrees n, regardless of divisibility by $gcd(m, d)$. In both cases, the sufficiently large assumption on n is necessary; we illustrate this by contriving conditions under which C has only finitely many degree n points, extending results previously known in the case of hyperelliptic curves.

20th August
11:00am
Room I

The Fermat equation over real biquadratic fields

Maleeha Khawaja
University of Sheffield

Work of Freitas and Siksek provides a framework for attacking the Fermat equation over totally real fields via the modular approach. In this talk, we consider the Fermat equation $x^p + y^p = z^p$ over real biquadratic fields (e.g. $\mathbb{Q}(\sqrt{2}, \sqrt{3})$, where p is a rational prime. We also look at the challenges that arise when applying this framework to such fields.

20th August
10:00am
Room II

Brauer Relations and Isogenies between Jacobians of curves

Alexandros Konstantinou
UCL

The main purpose of this talk is to present how one might use representation theoretic machinery to verify classical isogenies between Jacobians of curves, and apply it to recover known theorems related to the parity conjecture. At the heart of all of this lies the study of relations between permutation representations of finite groups. This presentation will heavily rely on examples, where we will see how one utilises such relations to establish isogenies between Jacobians of curves (i.e. p-isogeny between 2 elliptic curves).

19th August
3:00pm
Room I

On Galois-Gauss sums for weakly ramified characters

Yu Kuang
King's College London

Let L/K be a weakly ramified Galois extension of number fields and set $G := Gal(L/K)$. When L/K is of odd degree, Bley, Burns and Hahn formulated a conjectural equality in the relative algebraic K-group that relates weakly ramified Galois-Gauss sums of the complex characters of G (twisted by the second Adams operator) and structural invariants of the square root of the inverse different of L/K. We investigate the possibility of formulating a generalization of the central conjecture of Bley, Burns and Hahn to the setting of all weakly ramified Galois extensions L/K in which a the square root of the inverse different exists.

Rational points and Selmer groups of genus 3 curves

18th August
11:00am
Room I

Jef Laga
University of Cambridge

Manjul Bhargava and Arul Shankar have determined the average size of the n-Selmer group of the family of all elliptic curves over $\mathbb{Q}$ ordered by height, for n at most 5. They used this to show that the average rank of elliptic curves is less than one. In this talk we will consider a family of nonhyperelliptic genus 3 curves, and bound the average size of the 2-Selmer group of their Jacobians. This implies that a majority of curves in this family have relatively few rational points. We also consider a family of abelian surfaces which are not principally polarized and obtain similar results.

Mertens' Theorems for Number Fields

20th August
10:00am
Room I

Ethan Lee
UNSW Canberra

To understand the distribution of prime numbers, Mertens' theorems were an important step forward. They were proved 22 years before a rigorous proof of the prime number theorem. Both results can be generalised into the number fields setting; these are called Mertens' theorems for number fields and the prime ideal theorem. All of the error terms in these results can be described explicitly, and in this talk, we will observe that there are distinct advantages to using either result in different situations. I will also introduce an application of Mertens' theorems for number fields pertaining to the number of factors of a polynomial.

A newform theory for Katz modular forms

18th August
2:00pm
Room I

Daniel Mamo
University of Addis Ababa

In this talk, a multiplicity one theorem for Katz modular forms is presented. We show that a cuspidal Katz eigenform which admits an irreducible residual Galois representation is in the level and weight old space of a uniquely associated Katz newform. We also set up variants of multiplicity one results for Katz eigenforms which have reducible residual Galois representation.

20th August
10:30am
Room I

A Unique Perfect Power Decagonal Number

Philippe Michaud-Rodgers
University of Warwick

Polygonal numbers, perfect powers, and the relationships between them have been studied since the time of Pythagoras. Let $D(n) = 4n^2 - 3n$ denote the nth decagonal number. For which values of n is $D(n)$ a perfect power? To answer this question, we will construct various elliptic curves, called Frey curves, and see how they are connected to certain modular forms using the theory of modular curves, mimicking the proof of Fermat's Last Theorem. We prove that $D(3) = 3^3$ is the only perfect power decagonal number > 1. The aim of this talk is to serve as an introduction to using the modular method for resolving explicit Diophantine equations. We will discuss some difficulties that can arise, and how to try and overcome them.

19th August
10:30am
Room III

Class group of real cyclotomic fields

Mohit Mishra
Harish-Chandra Research Institute

In 1961, Cornell proved that every finite abelian group occurs as a subgroup of class group of some cyclotomic field. There is no analogous result known for totally real number fields. In this talk, we will prove that every finite abelian group occurs as a subgroup of the class group of infinitely many real cyclotomic fields. This is a joint work with Prof. L. C. Washington and Prof. R. Schoof.

19th August
10:30am
Room I

p-adic denseness of fractions of values of polynomials with integer coefficients

Piotr Miska
Jagiellonian University in Krakow

For any set $A \subset \mathbb{Z}$ we define its ratio set as

$$R(A) = \{a/b : a, b \in A, b \neq 0\}.$$

The topic of denseness of sets $R(A)$ in positive real half-line generated a lot of literature. Recently, the subject of denseness of these sets in p-adic fields is also under investigation and, as a new area of study, has many open problems. The talk is devoted to one of them. Let $f \in \mathbb{Z}[X]$ be fixed. Then, one can ask how large is the set

$$\mathbb{P}_f = \{p \in \mathbb{P} : R(f(\mathbb{Z})) \text{ is dense in } \mathbb{Q}_p\}.$$

During the talk we will show that if f is irreducible over $\mathbb{Z}$, then the set $\mathbb{P}_f$ has the asymptotic density $d_{\mathbb{P}}(\mathbb{P}_f)$ with respect to the set of prime numbers and this density is positive. Moreover, we will focus on the set of all the possible values of $d_{\mathbb{P}}(\mathbb{P}_f)$, when $f \in \mathbb{Z}[X]$ runs over all the irreducible polynomials over $\mathbb{Z}$.

Formalization of p-adic L-functions in Lean

18th August
2:30pm
Room II

Ashvni Narayanan

Imperial College London

I am formalizing p-adic L-functions in an automated theorem prover called Lean. The definitions are optimized to move towards formalizing the statement of the Iwasawa Main Conjecture. One must also ensure that all theorems are proved in the utmost generality. We use a non canonical definition in terms of the p-adic integral, and use it to prove some properties.

Fekete Polynomials, Quadratic Residues, and Arithmetic

20th August
2:30pm
Room I

Tung Nguyen

Western University

Fekete polynomials play an important role in the study of special values of L-functions. While their analytic properties are well-recorded in the literature, little is studied about their arithmetic. In this talk, we plan to explain some surprising properties of these polynomials. In particular, we will see that Fekete polynomials contain some rich arithmetic information. This is based on joint work with Jan Minac and Nguyen Duy Tan.

Average Ranks of Elliptic Curves (over p-extensions)

20th August
11:00am
Room III

Ross Paterson

University of Glasgow

As E varies among elliptic curves defined over the rational numbers, a theorem of Bhargava and Shankar shows that the average rank of the Mordell–Weil group $E(\mathbb{Q})$ is bounded. If we now fix a number field K, is the same true of $E(K)$? If so, then how does the average rank depend on (and grow with) K? This talk will report on recent progress on these questions: answering the first in the affirmative for certain choices of K, and offering a partial answer to the second.

20th August
2:30pm
Room II

The p-adic Section Conjecture and valuations

Jesse Pajwani
Imperial College London

The p-adic section conjecture is a long standing conjecture about curves of high genus over p-adic fields. It says that if k is a p-adic field and X is a curve of genus ≥ 2 over k, then we can link $X(k)$ to sections of a short exact sequence of étale fundamental groups. One of the strongest results made towards proving the section conjecture was a result of Pop and Stix, which linked each section of this short exact sequence to a valuation on the function field of the curve. The original proof is quite involved, however, there is an alternative, combinatorial proof. I'll first introduce the section conjecture, then give a (very rough) sketch of the combinatorial proof of this theorem, and if there's any time left, I'll hopefully talk about where this leaves us.

19th August
10:30am
Room II

Entanglement in the family of division fields of a CM elliptic curve

Riccardo Pengo
École normale supérieure de Lyon

The entanglement in a family of number fields "measures" to what extent the family fails to be linearly disjoint. In this talk, based on a joint work with Francesco Campagna, I will explain why the entanglement in the family of number fields generated by the torsion points of a CM elliptic curve is very small, in a precise, explicitly quantifiable sense. Moreover, I will give a necessary condition to have a non-trivial entanglement, and I will describe in detail the entanglement for elliptic curves defined over the rational numbers, which always satisfy this condition.

18th August
2:00pm
Room II

Iwasawa theory over three-dimensional p-adic Lie extensions

Chao Qin
Sun Yat-sen University

Iwasawa theory is a powerful tool which describes the mysterious relationship between arithmetic objects and the special values of L-functions. A precise form of this relationship is neatly encoded in the so-called "Iwasawa Main Conjecture". In this talk, I will introduce Main Conjecture in the non-commutative setting. Then I will describe the $K_1(\mathbb{Z}_p[[G_\infty]])$ and its localizations using p-adic congruences, where G_∞ is any p-adic Lie group with dimension 3.

Constructing Galois representations ramified at one prime

19th August
10:00am
Room III

Anwesh Ray
UBC, Vancouver

Given a prime p and a positive integer n satisfying further constraints, we construct a Galois representation with big image in $GL_n(\mathbb{Z}_p)$ which is unramified away from p. The construction applies to regular primes as well as irregular primes subject to a constraint of the index of irregularity. Such representations are constructed by lifting suitable residual representations with image in the diagonal torus of $GL_n(\mathbb{Z}/p\mathbb{Z})$, for which the associated deformation problem is unobstructed.

Gluing curves of genus 1 and 2 along their 2-torsion

18th August
2:30pm
Room III

Sam Schiavone
MIT

Let X and Y be curves of genus 1 and 2, respectively, over a base field k of characteristic not 2. In this talk, we give criteria for the existence of a curve Z over k whose Jacobian is (up to twist) $(2,2,2)$-isogenous to the product of the Jacobians of X and Y. Moreover, we present an algorithm to compute equations for the curve Z given equations for X and Y using the geometry of Kummer surfaces.

Non-monogenic Division Fields and Endomorphisms of Abelian Varieties

18th August
2:00pm
Room III

Hanson Smith
University of Connecticut

A number field is said to be monogenic if its ring of integers admits a power integral basis. In this talk we will detail recent work constructing a matrix that yields an explicit description of the Frobenius endomorphism of certain abelian varieties. We will then outline how this matrix is employed in a new algorithm that detects local obstructions to the monogeneity of the division fields of these abelian varieties.

20th August
2:00pm
Room III

Groups of rational points of abelian varieties over finite fields

Caleb Springer
The Pennsylvania State University

We will show that every finite abelian group occurs as the group of rational points of an ordinary abelian variety over the finite field with two elements. Similar results hold over finite fields of larger cardinality. On our way to proving these results, we will view the group of rational points of an abelian variety as a module over its endomorphism ring. By describing this module structure in important cases, we obtain (a fortiori) an understanding of the underlying groups. Combining this description of structure with recent results on the cardinalities of groups of rational points of abelian varieties over finite fields, we will deduce the main theorem. This work is joint with Stefano Marseglia.

19th August
10:00am
Room II

Local Galois module theory: An overview

Lorenzo Stefanello
Università di Pisa

The study of the Galois module structure of the valuation ring of a p-adic field has always been a very interesting problem in number theory, and it has stimulated the minds of many great mathematicians. In this talk, we see an exposition of the main results of this theory, with a special focus on a more modern approach, which makes use of Hopf–Galois theory. In the final part, we also mention some connections with other areas, such as group theory, and we briefly discuss about a joint work with Andrea Caranti.

20th August
10:00am
Room III

Campana points and powerful norms

Sam Streeter
University of Bath

The theory of Campana points is of growing interest in arithmetic geometry due to its ability to interpolate between the notions of rational and integral points. Further, it naturally lends itself to studying "arithmetically interesting" solutions of equations. In this talk, I will introduce Campana points and explain the key ideas and principles behind recent results on asymptotics for Campana points of bounded height, providing evidence for a Manin-type conjecture proposed in work of Pieropan, Smeets, Tanimoto and Várilly-Alvarado. I will also indicate how these results give rise to an asymptotic formula for powerful (e.g. square-full) values of norm forms.

Note on the p-divisibility of class numbers of an infinite family of imaginary quadratic fields

18th August
10:30am
Room II

Pasupulati Sunil Kumar
IISER TVM

Let K be a number field. The ideal class group Cl_K is defined to be the quotient group J_K/P_K, where J_K is the group of fractional ideals of K and P_K is the group of principal fractional ideals of K. It is well known that Cl_K is finite. The class number h_K of a number field K is the order of Cl_K. The ideal class group is one of the most basic and mysterious objects in algebraic number theory. The divisibility properties of the class number of number fields play a significant role in understanding the structure of the ideal class groups of number fields. I will talk about p-divisibility of the class number of new family $\mathbb{Q}(\sqrt{1-2m^p})$, where m is the power of odd prime and p is an odd prime. In the end, I will also talk about Iizuka's conjecture in a particular case. This is joint work with Srilakshmi Krishnamoorthy.

Realizing Galois representations in Mordell–Weil groups of hyperelliptic curves

20th August
2:00pm
Room II

Arvind Suresh
University of Georgia

Let L/K be a Galois extension of number fields, with group G. Consider the following problem: given a finite $\mathbb{Q}[G]$-module V and integer g, construct a g-dimensional abelian variety J/K such that $J(K)$ contains V. We show by explicit construction that if g is large enough, then there exist infinitely many genus g hyperelliptic curves X/K, with Jacobian J/K absolutely simple, and such that $J(K)$ contains V. Here are two examples when J/K is an elliptic curve: (a) L/K is cyclic of degree 16, and V is the unique 8-dim'l irrep. of G; (b) L/K is cubic cyclic, and V consists of four copies of the two-dim'l irrep. of G. We obtain our results by combining and building on work of Mestre–Shioda and Liu–Lorenzini.

Selmer Spaces and (Derived) Symplectic Geometry

18th August
10:30am
Room I

Jay Swar
University of Oxford

Selmer groups of an elliptic curve were generalized to any Galois representation by Bloch and Kato; a further generalization to a mildly non-abelian cohomology set was obtained by Kim. This set is in fact the points of a scheme which has notably been exploited for effective Mordell-Faltings applications. We'll investigate a different aspect of these spaces, establishing some nice descriptions of their cotangent bundles, a geometric interpretation of some arithmetic dualities, and the application of a powerful theorem coming from (derived) symplectic geometry.

19th August
11:00am
Room II

A Berkovich-analytic approach to models of curves over DVRs

Daniele Turchetti
University of Warwick

Let K be a complete discrete valuation field and let R be its ring of integers. For a smooth projective curve X over K there are many curves over R that become isomorphic to X after base-changing to K. Those that are flat and proper are called 'models' of X. The theory of models is rich of applications to topics of arithmetic interest, such as Diophantine geometry, Galois representations, and cryptography. In this talk, I will present two results on the behaviour of models of curves under base change. The first (joint with Lorenzo Fantini) exploits the geometry of the Berkovich analytification of X to relate certain regular models with the minimal extension L/K necessary for X to become semi-stable; the second (joint with Andrew Obus) is a study of models of curves with potentially multiplicative reduction.

18th August
10:00am
Room I

Semistable reduction and tame ramification

Art Waeterschoot
KU Leuven

Every nice curve over a local field is potentially semistable, meaning that we can find a nice model over the ring of integers if we allow a finite extension of the base field. Using Berkovich geometry, we will explain how to find this finite extension in case a tamely ramified extension suffices.

20th August
2:00pm
Room I

From Birch and Swinnerton-Dyer to Bloch-Kato conjecture

Wojtek Wawrów
London School of Geometry and Number Theory

The Birch and Swinnerton-Dyer conjecture is one of the most famous open problems in number theory. As it turns out, it is only a very small piece of a much broader statement, known as the Bloch-Kato conjecture. In this talk we will discuss the representation-theoretic interpretation of the BSD conjecture and how it fits into this broader context, as well as what work has been done towards the Bloch-Kato conjecture, including some recent and future work.

Elements of prime order in Tate-Shafarevich groups of abelian varieties over $\mathbb{Q}$

20th August
10:30am
Room II

Ariel Weiss

The Hebrew University of Jerusalem

Very little is known about the Tate-Shafarevich groups of abelian varieties. On the one hand, the BSD conjecture predicts that they are finite. On the other hand, heuristics suggest that, for each prime p, a positive proportion of elliptic curves $E/\mathbb{Q}$ have $Ш(E)[p] \neq 0$, and one expects something similar for higher dimensional abelian varieties as well. Yet, despite these expectations, it seems to be an open question whether, for each prime p, there exists even a *single* elliptic curve over $\mathbb{Q}$ with $Ш(E)[p] \neq 0$. In this talk, I will prove that, for each prime p, there exists a geometrically simple abelian variety $A/\mathbb{Q}$ with $Ш(A)[p] \neq 0$. Our examples arise from Eisenstein quotients of modular Jacobians. This is joint work with Ari Shnidman.

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