## Y-RANT 2023

## Conference Booklet

Organisers:<br>Sven Cats and Lukas Kofler



Venue. The conference is being hosted at the Centre for Mathematical Sciences (CMS) at the University of Cambridge, on Wilberforce Road (CB3 0WA). Talks will take place in lecture theatres MR2, MR3 and MR9.

Accommodation. Most participants will stay in a single room in Churchill College, Storey's Way (CB3 0DS). It is a 45 minute walk through the town centre from the train station, or accessible by bus (see below).

Conference Dinner. The conference dinner will be held at $7: 30 \mathrm{pm}$ on Wednesday the 6 th of September in the Hall of Peterhouse on Trumpington Street (CB2 1RD). It is a 25 minute walk from Churchill College and from the CMS, and accessible by bus (see below).

Travel. Most locations in Cambridge are accessible by foot. Alternatively, you can travel by bus as explained on the following website: https://www.environment.admin.cam.ac.uk/travel/travel-bus

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| Wednesday 6th September |  |  |
| :---: | :---: | :---: |
| 09:00-9:30 | Registration |  |
| 9:30-10:30 | Plenary <br> The curves | Jan Vonk |
| 10:30-11:00 | Coffee Break |  |
| 11:00-11:30 | Yubo Jin <br> Special L-values for Classical Groups | Alberto Angurel Andres <br> On the structure of the Selmer group of elliptic curves |
| 11:30-12:00 | Aleksander Horawa Algebraic cycles and the Langlands program | Alvaro Gonzalez Hernandez <br> Crazy for two: Genus 2 curves in characteristic 2 via Kummer surfaces |
| 12:00-12:30 | $\begin{aligned} & \text { Arshay Sheth } \\ & \text { Introduction to Explicit } \\ & \text { Reciprocity Laws } \end{aligned}$ | Art Waeterschoot <br> Jumps of Jacobians via orthogonal differential forms |
| 12:30-14:30 | Lunch Break |  |
| 14:30-15:00 | Arun Soor <br> Quasicoherent sheaves for rigid analytic geometry | Caleb Springer <br> Decidability, definability, and groups of units in totally imaginary fields |
| 15:00-15:30 | $\qquad$ <br> Skew Hermitian Forms over CM fields with complex conjugation | Diana Mocanu <br> The modular approach for solving $x^{r}+y^{r}=z^{p}$ over totally real number fields |
| 15:00-15:30 | Constantinos Papachristoforou <br> Representation theory of p-adic <br> groups | Elvira Lupoian <br> Rational Cuspidal Points on Modular Jacobians |
| 16:00-16:30 | Coffee Break |  |
| 16:30-17:00 | Daniel Puignau <br> Noncommutative Euler Systems | Brauer-Manin obstruction for K3 <br> surfaces |
| 17:00-17:30 | Harry Spencer Motivic pieces of curves | Isabel Rendell <br> p-adic heights on elliptic curves |


| Thursday 7th September |  |  |
| :---: | :---: | :---: |
| 9:30-10:30 | Plenary: Celine Maistret <br> The Birch-Swinnerton-Dyer conjecture and the parity conjecture |  |
| 10:30-11:00 | Coffee Break |  |
| 11:00-11:30 | Håvard Damm-Johnsen Modular algorithms for Gross-Stark units and Stark-Heegner points | $\square$ <br> Jamie Bell <br> Points on curves over dihedral extensions |
| 11:30-12:00 | Explicit images of Shimura's map | Newform Eisenstein congruences of local origin |
| 12:00-12:30 | James Kiln <br> An introduction to Eigencurves | Katerina Santicola Curves with prescribed rational points |
| 12:30-14:30 | Lunch Break |  |
| 14:30-15:00 | James Rawson <br> Computing Derivatives of Hecke Eigenvalues in p-Adic Families of Modular Forms | Marios Voskou <br> Hyperbolic Counting Problems |
| 15:00-15:30 | Tom Adams <br> The Drinfeld Upper Half Plane | Mohamed Tawfik <br> Transcendental Brauer groups of Kummer surfaces, and Brauer-Manin obstruction |
| 15:30-16:00 | Mark Heavey <br> Picard groups of p-adic affinoid <br> algebras | Pedro José Cazorla García On differences of perfect powers and prime powers |
| 16:00-16:30 | Coffee Break |  |
| 16:30-17:00 | Rafail Psyroukis <br> On a Rankin-Selberg integral of three Hermitian cusp forms and its relation with the twisted spinor L-function | Robin Visser <br> Abelian surfaces with good reduction away from 2 |
| 17:00-17:30 | Zachary Feng <br> p-adic L-functions and the GL(1)-eigenvariety | Tim Santens <br> Integral points of bounded height on toric varieties |


| Friday 8th September |  |  |
| :---: | :---: | :---: |
| 09:30-10:00 | Marti Oller Riera <br> Squarefree values of discriminant polynomials | Harvey Yau Brauer elements on elliptic surfaces |
| 10:00-10:30 | James Taylor <br> Geometry of Drinfeld Spaces | Sam Frengley <br> On the geometry of the Humbert surface of discriminant $N^{2}$ |
| 10:30-11:00 | Coffee Break |  |
| 11:00-11:30 | $\square$ <br> Vincenzo di Bartolo <br> From the BT-building of $\mathrm{SL}_{n}(K)$ to its smooth representations | Abdulmuhsin Alfaraj <br> Manin's conjecture for equivariant compactifications of vector groups |
| 11:30-12:00 | Radu Toma <br> On the size of automorphic forms of large level on PGL( $n$ ) | Corijn Rudrum <br> On the method of Chabauty and Coleman |
| 12:00-12:30 | Mads Christensen Special cycles on arithmetic hyperbolic manifolds | Yan Yau Cheng <br> Arithmetic Triple Linking Numbers |
| 12:30-14:30 | Lunch Break |  |
| 14:30-15:30 | Plenary: <br> What is I | David Loeffler |

## Jan Vonk <br> The curves of Diophantus

Abstract: This is the story of an old equation of Diophantus, which will take us on an example-based excursion around certain questions about torsion subgroups of elliptic curves. We will also discuss the singularly fruitful role played by the theory of complex multiplication in these investigations.

## Celine Maistret <br> The Birch-Swinnerton-Dyer conjecture and the parity conjecture

Abstract: Let $K$ be a number field. The foundational Mordell-Weil theorem asserts that the $K$-rational points of an abelian variety over $K$ form a finitely generated abelian group, the Mordell-Weil group. Systematically computing the rank of this group proved very challenging and is predicted by the Birch-Swinnerton-Dyer conjecture to be equal to the order of vanishing of the $L$-function of the variety at a special value. While theoretical and numerical evidence have been provided towards this conjecture for elliptic curves, the case of higher dimension remains virtually untouched.

In this talk we will discuss some evidence that the Birch-Swinnerton-Dyer correctly predicts the parity of the rank of certain abelian surfaces. To do so, I will recall the origin and statements of the Birch-SwinnertonDyer conjecture, and introduce the parity conjecture. I will discuss the methods used to prove some selected instances of the parity conjecture and their current limitations. Most of these limitations come from challenges around the local arithmetic of abelian varieties which I will present.
David Loeffler
What is Iwasawa Theory?

Abstract: I will introduce some of the basic tools and ideas of Iwasawa theory, and how it is used to shed light on the relation between special values of $L$-functions and algebraic objects, such as elliptic curves and class groups of number fields.

## Yubo Jin <br> Special L-values for Classical Groups

Abstract: In this talk, I will present an integral representation of $L$-functions for classical groups using the doubling method. I will also briefly introduce the applications of the integral representation on proving Deligne's conjecture and constructing $p$-adic $L$-functions.

## Aleksander Horawa <br> Algebraic cycles and the Langlands program

Abstract: The Langlands program interacts with the theory of algebraic cycles in several ways. The Hodge and Tate conjectures predict that some instances of Langlands functoriality are realized by algebraic cycles. Conjectures on special values of $L$-functions suggest that higher Chow groups are related to the contributions to cohomology of automorphic forms, leading to the "motivic action conjectures". We will give a broad overview of the subject, focusing on examples, without assuming prior knowledge of any of the above concepts.

## Introduction to Explicit Reciprocity Laws

Abstract: In contemporary terminology, explicit reciprocity laws often refer to relations between values of $L$-functions and Euler systems, and have been a key tool to prove certain cases of the Birch and Swinner-ton-Dyer conjecture. Nevertheless, they have a very rich history going all the way back to the beginnings of class field theory. In this talk, we will discuss how the concept of explicit reciprocity laws has evolved through time and end by explaining some important developments that have taken place in the subject in recent years.

## Arun Soor <br> Quasicoherent sheaves for rigid analytic geometry

Abstract: Let $X$ be a rigid analytic variety. The example of Gabber shows that the naive definition of quasicoherent sheaf on $X$ does not satisfy Cartan's theorems A and B. On the other hand, the work of Kremnizer et al shows that derived descent results are possible. Borrowing ideas from derived analytic geometry, we will show that the functor sending $\operatorname{Sp}(A)$ to the underlying $\infty$-category of (unbounded) chain complexes of $A$-modules, is a sheaf in the analytic topology. We will introduce some basic operations on these "quasicoherent sheaves", including pullback, pushforward and shriek pullback.

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\begin{array}{|l|} 
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\\
\text { Skew Hermitian Raj Bhatta } \\
\text { Forms over CM fields with complex conjugation }
\end{array}
$$

Abstract: We study lattices with skew Hermitian forms over division algebras with positive involutions. For centre field of division algebras of Albert types IV which are precisely CM fields, we show that such a lattice contains an "orthogonal" basis for a sublattice of effectively bounded index. Second, we apply this result to obtain new results in the field of unlikely intersections. More specifically, we prove the Zilber-Pink conjecture for the intersection of curves with special subvarieties of simple PEL type IV (restricted to CM field) under a large Galois orbits conjecture. This result extends on the paper by Dr M Orr and Dr C Daw who solved Type I and Type II under a large Galois orbits conjecture.

## Constantinos Papachristoforou Representation theory of p-adic groups

Abstract: Driven by the Langlands program, the representation theory of reductive $p$-adic groups has been significantly developed during the last few decades. I will give an overview on various aspects of the theory, with particular emphasis on decomposition of categories. I will also discuss passing from the classical case of complex representations to more general coefficient rings.

## Daniel Puignau <br> Noncommutative Euler Systems

Abstract: Euler systems, introduced by Kolyvagin in the 90s, are algebraic-number theoretic objects which notably simplified the Iwasawa Main Conjecture (IMC) proof by Mazur and Wiles. Recently extended to the noncommutative setting by Burns-Sakamoto-Sano, they hold promise in illuminating the Noncommutative Class Number Formula conjecture, higher rank noncommutative IMCs, and enhancing our understanding of the far-reaching equivariant Tamagawa Number Conjecture, which in its most general form includes the Birch and Swinnerton-Dyer conjecture.

## Harry Spencer

Abstract: Inspired by recent work of Constantinou-Dokchitser-Green-Morgan on parities of ranks of Jacobians of curves, we discuss an analogue of Artin-twisted Hasse-Weil $L$-functions. In this down-to-Earth talk we work mostly with explicit examples, touching on motives with (non-commutative) coefficients, analogues of BSD and numerical computation. The small number of original observations are from joint work with Giorgio Navone and Corijn Rudrum.

## Håvard Damm-Johnsen <br> Modular algorithms for Gross-Stark units and Stark-Heegner points

Abstract: In the last few decades, many attempts have been made to extend CM theory to the setting of real quadratic fields. In this talk I will describe how to turn recent work by Darmon, Pozzi and Vonk into an efficient algorithm for computing $p$-adic analogues of modular units and Heegner points: Gross-Stark units and Stark-Heegner points. This builds on the framework of Darmon and Vonk's rigid meromorphic cocycles, and involves Hilbert modular forms, overconvergent elliptic modular forms, and Gauss' reduction theory of binary quadratic forms.

## James Branch <br> Explicit images of Shimura's map

Abstract: In 1973, in a momentous paper, Shimura introduced a map $\sigma_{D}$ between half integral weight and integral weight modular forms. Since then, motivated by an unpublished work of Selberg, Cipra (1987) gave explicitly the Shimura image of $f\left(4 r_{\psi} z\right) h_{\psi}(z)$, an old Hecke eigenform times a theta function $h_{\psi}$ where $\psi$ has conductor a prime power $r_{\psi}$. Hansen and Naqvi generalised this to allow for arbitrary conductor $r_{\psi}$. In this talk we will discuss current progress in this direction.

## James Kiln <br> An introduction to Eigencurves

Abstract: In 1998, Coleman and Mazur constructed two rigid analytic spaces called eigencurves to parameterise spaces of finite slope p-adic overconvergent eigenforms. Soon after, Kisin used his finite slope subspace construction to allow him to prove certain cases of the Fontaine-Mazur conjecture; these methods later gave rise to the notion of triangulline Galois representations. In this talk I will give an introduction to eigencurves and explain a conjecture of Kisin, later proven by Emerton, that the finite slope subspaces are isomorphic to component parts of Coleman and Mazur's eigencurve.

# James Rawson <br> Computing Derivatives of Hecke Eigenvalues in p-Adic Families of Modular Forms 

Abstract: By work of Hida and Coleman, it is known that all eigenforms vary in $p$-adic families. Informally, such a family can be thought of a $q$-expansion, where the coefficients are $p$-adic power series such that specialising at a positive integer gives a modular form of that weight. Explicit descriptions of these families are, unfortunately, difficult to come by. In this talk, I will present a method for computing the derivative of Hecke eigenvalues in the family passing through a suitable modular form.

## Tom Adams <br> The Drinfeld Upper Half Plane

Abstract: The Drinfeld upper half plane is a rigid analytic space that serves as a non-archimedean analogue of the complex upper half plane. It is of number theoretic importance because, for example, both the Jacquet-Langlands and local Langlands correspondences can be realised in the (compactly supported, $\ell$-adic, etale) cohomology of a system of coverings of the Drinfeld upper half plane called the Drinfeld tower. In this talk, I will introduce the Drinfeld upper half plane. If time permits, I will discuss why we hope to be able to realise the smooth representations of $\mathrm{GL}_{2}\left(\mathbb{Z}_{p}\right)$ in the 'cohomology of the Drinfeld tower'.
Picard groups of p-adic affinoid algebras

Abstract: Rigid geometry is a branch of number theory concerned with extending the theory of complex analytic spaces to the case where the base field has a non-archimedean function. Rigid analytic spaces are locally ringed spaces with respect to an appropriate Grothendieck topology, thus meaning they have Picard groups. We present a result about the finite generation of the Picard group for an affinoid space (similar to affine schemes in scheme theory) under good enough regularity conditions, and outline possible future research on this topic.

## Rafail Psyroukis

On a Rankin-Selberg integral of three Hermitian cusp forms and its relation with the twisted spinor L-function
Abstract: Let $K=\mathbb{Q}(i)$. In this work, we examine the Petersson inner product of a Hermitian Eisenstein series of Siegel type on the unitary group $U_{5}(K)$, diagonally-restricted on $U_{2}(K) \times U_{2}(K) \times U_{1}(K)$, against two Hermitian cusp forms $F, G$ of degree 2 and a Hermitian cusp form $h$ of degree 1 , all having weight $k \equiv 0$ $(\bmod 4)$. This consideration gives an integral representation of a certain Dirichlet series, which will then have an analytic continuation and functional equation, due to the one of the Eisenstein series. By taking $F$ to belong in the Maass space, we are able to show that the Dirichlet series possesses an Euler product. Moreover, its $p$-factor for an inert prime $p$ can be essentially identified with the degree $12 p$-factor of $Z_{G \otimes h}$, the spinor $L$-function attached to $G$, twisted by the Satake parameters of $h$. The question of whether the same holds for the primes that split remains unanswered here, even though we make considerable steps in that direction too. This joint work with Prof. Athanasios Bouganis is inspired by a paper of Heim, who considered a similar question in the case of Siegel modular forms.

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\begin{gathered}
\text { Zachary Feng } \\
\text { p-adic L-functions and the GL(1)-eigenvariety }
\end{gathered}
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Abstract: In Tate's thesis, he constructs Hecke $L$-functions by constructing a meromorphic function on the complex manifold of Hecke characters, whose restriction to certain 1-dimensional families give rise to Hecke $L$-functions. In this expository talk, we explore the analogy between Tate's construction and the construction of $p$-adic $L$-functions. Namely, we construct a rigid meromorphic function on the space of all $p$-adic Hecke characters, i.e. the GL(1)-eigenvariety, whose restriction to 1-dimensional families give rise to $p$-adic $L$-functions.

## Marti Oller Riera <br> Squarefree values of discriminant polynomials

Abstract: A significant open problem is to determine the density of squarefree values of polynomials, i.e. the "probability" that a multivariable integral polynomial $f\left(x_{1}, \ldots, x_{n}\right)$ takes a squarefree value. Granville and Poonen determined this density conditional to the ABC conjecture, but the general problem remains known only for some specific polynomials. The purpose of this talk is to introduce a paper by Bhargava, Shankar and Wang in which they determine the density of polynomials having squarefree discriminant, and we will also talk about how their results can be extended to more general families of discriminant polynomials.

## Geometry of Drinfeld Spaces

Abstract: The Drinfeld upper half plane and its covering spaces provide a geometric realisation of both the local Langlands correspondence and Jacquet-Langlands correspondence for $\mathrm{GL}_{2}(F)$, where $F$ is a finite extension of $\mathbb{Q}_{p}$. In this short talk we give an overview of the geometry of the upper half plane and the first covering space, their relationship with Deligne-Lusztig varieties, and discuss geometric invariants associated to these spaces.

# Vincenzo di Bartolo <br> From the BT-building of $\mathrm{SL}_{n}(K)$ to its smooth representations 

Abstract: TBA

On the size of automorphic forms of large level on $\operatorname{PGL}(n)$
Abstract: The sup-norm problem is part of the theory of arithmetic quantum chaos and it asks more precisely about the size of automorphic forms: viewed as waves, what can we say about their amplitude? In this talk, we discuss some new results in the level aspect of this problem. The focus lies on the counting problem at the heart of the matter and on understanding level structures in higher rank, both intimately tied to the theory of lattices and geometry of numbers.

## Mads Christensen <br> Special cycles on arithmetic hyperbolic manifolds

Abstract: Arithmetic hyperbolic manifolds are quotients of hyperbolic space by arithmetic groups. The best known examples are modular curves and Bianchi manifolds. These manifolds contain certain special cycles which are known to have modular properties. I will explain the meaning of this statement using explicit examples involving Heegner points and closed geodesics on modular curves. If time permits I will indicate some new ways in which these results could potentially be generalized.

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\begin{aligned}
& \quad \text { Alberto Angurel Andres } \\
& \text { On the structure of the Selmer group of elliptic curves }
\end{aligned}
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Abstract: Understanding the Selmer group of an elliptic curve has a crucial importance in the study of the Birch and Swinnerton-Dyer conjecture. In this talk, I will show how the structure of the p-Selmer group of an elliptic curve can be related to the modular symbols associated to the elliptic curve. For that purpose, we will take advantage of the Iwasawa theory of elliptic curves.

## Alvaro Gonzalez Hernandez <br> Crazy for two: Genus 2 curves in characteristic 2 via Kummer surfaces

Abstract: How can we do computations with the Jacobian of a genus 2 curve defined over a field of characteristic 2? Over fields of characteristic zero and odd, explicit models of the Jacobian as projective varieties have been computed but adapting them to the characteristic 2 case is far from straightforward. In this talk, I will discuss how this can be achieved through the study of the Kummer surface, which is a quotient of the Jacobian by an action that fixes the 2 -torsion points.

## Art Waeterschoot <br> Jumps of Jacobians via orthogonal differential forms

Abstract: Given a nice curve $C$ over a discretely valued field $k$, one can measure how a differential form degenerates on integral models; this yields a valuation on the vector space of holomorphic forms on $C$. It turns out that if one evaluates an orthogonal basis, one recovers Edixhoven's jumps, these are arithmetic invariants measuring the change of Néron models of $\operatorname{Jac}(C)$ under tame base change. I will explain how this can be used to compute the jumps in a rich class of examples. Joint with Michaël Maex and Enis Kaya.

## Decidability, definability, and groups of units in totally imaginary fields

Abstract: One of the classic tasks of number theory, known formally as Hilbert's Tenth Problem, is the following: Given any multivariable polynomial equation with integer coefficients, decide whether that equation has an integer solution. Famously, the DPRM theorem in 1970 showed that there is no algorithm which solves this problem, and we say that Hilbert's Tenth Problem is undecidable. More generally, using terminology from logic, we can consider the algorithmic decidability of the existential or first-order theories of a ring of arithmetic interest. In this talk, we will define these words and present recent results on first-order undecidability for rings of integers in certain infinite totally imaginary extensions of the rational numbers. The key technique will demonstrate existential definability for the maximal totally real subring by exploiting the unit groups of non-maximal orders.

## Diana Mocanu <br> The modular approach for solving $x^{r}+y^{r}=z^{p}$ over totally real number fields

Abstract: Since Wiles' famous proof of Fermat's Last Theorem, number theorists extensively studied Diophantine equations using the modular approach. We will briefly describe a variation of this method using (partial results about) modularity of elliptic curves over totally real fields, image of inertia comparison, and the study of certain $S$-unit equations.

Then, we will describe how to attack the equation $x^{r}+y^{r}=z^{p}$ (fixed $r$, varying $p$ ) using the above method. If time permits, we will sketch how to attack the very similar family of equations $x^{p}+y^{p}=z^{2}$ and $x^{p}+y^{p}=z^{3}$ (varying p ).

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\text { Rational } \begin{array}{|l|}
\hline \text { Elvira Lupoian } \\
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\end{array}
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Abstract: For a fixed prime $p \geq 5, \mathrm{Ogg}$ proved that the subgroup generated by the two cuspidal points of the modular curve $X_{0}(p)$ generate a cyclic subgroup of the Jacobian $J_{0}(p)$, of order the numerator of $p-1 / 12$. Mazur later proved that this is in fact the entire rational torsion subgroup of $J_{0}(p)$. The theorems of Manin and Drinfeld tell us that for any modular curve, the difference of two cusps is a torsion point on its Jacobian. Thus, we may ask, abstractly, what the subgroup generated by cusps is. In this talk, we'll give an overview of some well-studied cases, as well as some new results for the intermediate modular curves $X_{H}(p)$, where $H$ is a subgroup of $(\mathbb{Z} / p \mathbb{Z})^{*}$.

## Giorgio Navone <br> Brauer-Manin obstruction for K3 surfaces

Abstract: The talk is a gentle introduction to the Brauer-Manin obstruction to the Hasse principle for K3 surfaces. In particular, I will recall the most important definitions, like the Brauer-Manin set and Skorobogatov's conjecture on K3 surfaces, followed by a general overview which motivates my current research work in this area. I will conclude proposing and discussing an independent open question whose solution would be extremely relevant to this topic.

## Isabel Rendell <br> p-adic heights on elliptic curves

Abstract: In this talk I will discuss a history of height functions on elliptic curves, starting with the more commonly known real-valued heights and then their development into $p$-adic-valued. The construction of $p$-adic heights can be approached in different ways - from the viewpoint of divisors of degree zero or of points on an elliptic curve. I will discuss some properties and, time permitting, also discuss some applications of $p$-adic heights, for example their use in proving the finiteness of the set of integral points on a rank 1 elliptic curve.

## Jamie Bell <br> Points on curves over dihedral extensions

Abstract: The parity conjecture is an example of how local data can tell us about the rational points on a curve. In my talk I will discuss how we can refine the techniques used to study the parity conjecture, and use local data about a curve over the rationals to tell us about the points on this curve over a dihedral extension of $\mathbb{Q}$.

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\text { Newform } \begin{array}{|l|}
\hline \text { Jenny Roberts } \\
\text { Eisenstein congruences of local origin }
\end{array}
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Abstract: The theory of Eisenstein congruences dates back to Ramanujan's surprising discovery that the Fourier coefficients of the discriminant function are congruent to the 11th power divisor sum modulo 691. This observation can be explained via the congruence of two modular forms of weight 12 and level 1 ; the discriminant function and the Eisenstein series, $E_{12}$. We explore a generalisation of this result to newforms of weight $k>2$, squarefree level and non-trivial character.

## Katerina Santicola <br> Curves with prescribed rational points

Abstract: Given a nonsingular curve $C / \mathbb{Q}$ with genus $\geq 2$, we know by Falting's Theorem that $C(\mathbb{Q})$ is finite. Determining $C(\mathbb{Q})$ is a difficult problem, as we have no effective results on computing this set. We ask the reverse question: given a finite set of rational points $S \subseteq \mathbb{P}^{n}(\mathbb{Q})$, does there exist a nonsingular curve such that $C(\mathbb{Q})=S$ ? In this talk I will outline how to construct a positive and effective answer to this question, highlighting some of the theorems and tricks that were useful along the way.

## Marios Voskou <br> Hyperbolic Counting Problems

Abstract: In Euclidean space the Gauss' Circle Problem is concerned with estimating the number of lattice points inside a Euclidean circle. In this talk we will discuss various analogous problems in the hyperbolic space. We will explain why these problems are much harder and interpret them in terms of the distribution of arithmetic objects such as sums of four squares and norms of ideals in number fields. We use techniques from the analytic theory of automorphic forms, demonstrating the duality between the discrete and the continuous.

## Mohamed Tawfik

Transcendental Brauer groups of Kummer surfaces, and Brauer-Manin obstruction
Abstract: Skorobogatov and Zarhin showed that for a Kummer surface $X$, the transcendental Brauer group is finite, hence, it suffices to study the $p$-primary parts for all primes $p$. In another paper, they set out to discuss the case when $X=\operatorname{Kum}\left(E x E^{\prime}\right)$, a Kummer of a product of elliptic curves. The case when $E$ and $E^{\prime}$ are CM elliptic curves is of particular interest because of the richness of their endomorphism groups. They studied the case when $E$ and $E^{\prime}$ have CM by $\mathbb{Z}[i]$. We embark on discussing other CM cases. Moreover, we give an example on CM by $\sqrt{-2}$. Further, when $E$ and $E^{\prime}$ have $C M$ by $\mathbb{Z}\left[\zeta_{3}\right]$, where $\zeta_{3}$ is a primitive cubic root of unity, we show that a 5 - or a 7 -torsion element of the transcendental Brauer group of $X$ always gives rise to Brauer-Manin obstruction to weak approximation on $X$. Given time, I will focus on the calculations of the Manin pairing that proves that a generator of the 5-part of the transcendental Brauer group gives rise to Brauer-Manin obstruction to weak approximation on $X$.

## Pedro José Cazorla García <br> On differences of perfect powers and prime powers

Abstract: In 2004, Mihăilescu proved that the only consecutive perfect powers are 8 and 9 . Despite many attempts to generalise this conjecture to perfect powers with arbitrary difference $D$, not much more is known today.

Given a squarefree integer $1 \leq C_{1} \leq 20$ and a prime $2 \leq q<25$, we will present a methodology that allows us to resolve the following Diophantine equation

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C_{1} x^{2}+q^{\alpha}=y^{n}
$$

therefore determining which integers with squarefree part $C_{1}$ are the difference of a perfect power and a $q$-power.

This methodology combines the modular method popularised after the proof of Fermat's Last Theorem with an improved Thue-Mahler solver and new estimates on lower bounds on linear forms in three logarithms.

## Robin Visser

Abelian surfaces with good reduction away from 2
Abstract: Let $K$ be a number field, $d$ a positive integer, and $S$ a finite set of places in $K$. One of the crowning achievements of 20th century arithmetic geometry was Faltings proof that there are only finitely many isomorphism classes of dimension $d$ abelian varieties $A / K$ with good reduction away from $S$. Whilst some algorithms have been developed to effectively classify all such abelian varieties in certain cases, even the problem of classifying all abelian surfaces over $\mathbb{Q}$ with good reduction away from 2 still appears out of reach. In this talk, I will present a method which combines a criterion of Faltings-Serre-Livné with a computation of the possible associated 2-adic Galois representations to compute all isogeny classes of abelian surfaces over $\mathbb{Q}$ with good reduction away from 2 , if one further assumes some conditions on the field of 2-torsion of such surfaces.

## Tim Santens <br> Integral points of bounded height on toric varieties

Abstract: The distribution of rational points on toric varieties has been studied by proving asymptotics for the number of them of bounded height for a variety of height functions by Batyrev and Tschinkel. Adapting their methods to prove similar results for integral points turned out to be not so simple. In this talk I will explain why integral points are quite a bit more difficult than rational points and discuss recent work where I resolve this difficulty.

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\frac{\text { Harvey Yau }}{\text { lements on elliptic surfaces }}
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Abstract: In this talk I will outline a method to construct Brauer elements of elliptic curves, and how it can be used to construct Brauer elements of surfaces with an elliptic fibration. Examples of 2-torsion elements will be given, as well as applications to Brauer-Manin obstructions on surfaces.

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\begin{aligned}
& \text { On the geometry of the Humbert surface of discriminant } N^{2}
\end{aligned}
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Abstract: The Humbert surface $\mathcal{H}_{N^{2}} \subset \mathcal{M}_{2}$ parametrises curves of genus 2 admitting an (optimal) morphism of degree N to an elliptic curve (equivalently, whose Jacobian is ( $N, N$ )-isogenous to a product of elliptic curves). We will describe how the surfaces $\mathcal{H}_{N^{2}}$ fit into the Enriques-Kodaira classification (building on work of Hermann when N is prime), their relation to Hilbert modular surfaces, and (if time permits) number theoretic questions regarding rational points and birational models over $\mathbb{Q}$.

## Corijn Rudrum <br> On the method of Chabauty and Coleman

Abstract: In this talk I will give an introduction to the Chabauty-Coleman method for computing the (finite) set of rational points on certain higher-genus curves over $\mathbb{Q}$. The origin of the method is an old result by Chabauty where he proves the Mordell conjecture for curves for which the rank of their Jacobian is less than their genus. Coleman later modified Chabauty's argument using his theory of integration on varieties over $p$-adic fields to obtain an effective method for computing the set of rational points on these curves. I will explain the general strategy of the method, and briefly introduce some of the concepts it relies on like rigid analytic spaces and Coleman integrals.

## Abdulmuhsin Alfaraj <br> Manin's conjecture for equivariant compactifications of vector groups

Abstract: Manin's conjecture gives a description of the distribution of rational points relative to an appropriate height function on Fano varieties over global fields. I will talk about the proof of Manin's conjecture for compactifications of a certain class of algebraic groups, namely $\mathbb{G}_{a}^{n}$. This method uses harmonic analysis techniques on the adelic points of the group, which have the structure of a locally compact group.

> Yan Yau Cheng Arithmetic Triple Linking Numbers

Abstract: Following the analogy between knots and primes introduced by Mazur in the 1960s, we define multiple linking numbers of primes in a number field $K$. I will outline the proof of a result by Amano et al. which relates mod 2 triple linking numbers (also sometimes called the Rédei symbol) to $L$-functions and modular forms, which gives an explicit and constructive example of the theorem by Weil-Langlands and Deligne-Serre. To conclude I will discuss some recent work-in-progress and difficulties encountered in trying to generalise this to mod 3 triple linking numbers.

## Participant list

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