Y-RANT VI

Conference Booklet

July 31st – August 2nd, 2024

Organizers: Arun Soor, Håvard Damm-Johnsen & Zachary Feng

Introduction & local information

Y-RANT VI will take place in the Mezzanine of the Mathematical Institute at the University of Oxford, on Woodstock Road. The 40 20-minute talks will be split into two concurrent tracks, one in Lecture Room 3 (L3) and one in the TCC seminar room. In-between, there will be breaks with coffee, tea and biscuits. On Thursday, after the second plenary talk, there will be a wine reception in the Mezzanine.

Many of the speakers will be lodged in St. Anne's college, which is a few minutes north of the Mathematical Institute along Woodstock Road.

We are grateful to the Heilbronn Institute of Mathematics, on behalf of the EPSRC, and to the London Mathematical Society for making Y-RANT VI possible through generous financial support. We also thank Oxford Maths for waiving the room booking fees, and the Clay Mathematics Institute for covering the costs of the wine reception.

Schedule

Wednesday 31. July		
9:00 - 10:00	Registration	
10:00 - 10:30	*Alexandros Groutides* When Rankin-Selberg periods meet Euler systems	*Alvaro Gonzalez Hernandez* K3 surfaces with everywhere good reduction
10:30 - 11:00	*Bence Hevesi* Local-global compatibility beyond the self-dual case	*Antau Yang* Continued Fractions in the Field of p-adic Numbers
11:00 - 11:30	Coffee break	
11:30 - 12:00	*Calle Sönne* Serre's Modularity conjecture	*Edison Au-Yeung* Introduction to divisibility sequences for elliptic curves with complex multiplication
12:00 - 14:00	Lunch	
14:00 - 14:30	*Catinca Mujdei* Kloosterman sums over reductive groups	*Harry Shaw* Diagonal del Pezzo surfaces of degree 2 with a Brauer-Manin obstruction
14:30 - 15:30	*Yiannis Fam* Quaternionic modular forms mod p	*Harry Spencer* Wild conductor exponents of trigonal and tetragonal curves
15:00 - 15:30	Coffee/tea break	
15:30 - 16:00	*David Kurniadi Angdinata* Denominators of BSD quotients	*Harvey Yau* Descent on elliptic surfaces and the Brauer-Manin obstruction
16:00 - 17:00	Plenary Talk: *Hanneke Wiersema* in L3	

Thursday 1. August		
9:30 - 10:00	*Frederick Thøgersen* A few remarks about p-adic L-functions	*Jamie Bell* Parity in dihedral extensions
10:00 - 10:30	*Ho Leung Fong* Cohomology of locally symmetric spaces	*Julie Tavernier* Counting number fields whose conductor is the sum of two squares
10:30 - 11:00	Coffee break	
11:00 - 11:30	*Jenny Roberts* Coefficients of modular polynomials (mod 23)	*Katerina Santicola* Computing the Brauer-Manin obstruction on del Pezzo surfaces of degree 4
11:30 - 12:00	*Vincenzo Di Bartolo* Augmented Iwasawa algebras in the Langlands program	*Kenji Terao* Faltings's Theorem and Isolated Points
12:00 - 14:00	Lunch	
14:00 - 14:30	*Mark Heavey* Picard groups of affinoid algebras	*Diana Mocanu* Variants of the modular method
14:30 - 15:00	*Martin Ortiz* Theta linkage maps and the weight part of Serre's conjecture	*Luca Ferrigno* Isogeny relations in products of families of elliptic curves
15:00 - 15:30	Coffee/tea break	
15:30 - 16:00	*Rafail Psyroukis* A Dirichlet series attached to modular forms of orthogonal groups	*Eda Kirimli* Isogeny graphs of Abelian surfaces
16:00 - 17:00	Plenary Talk: *Andrew Wiles* in L1	
17:00 - 18:00	Wine reception	

Friday 2. August		
9:30 - 10:00	*Simon Alonso* On the unramified Fontaine-Mazur and Boston conjectures	*Maryam Nowroozi* Perfect powers in elliptic divisibility sequences
10:00 - 10:30	*Siqi Yang* Hilbert modular forms and geometric modularity in quadratic case	*Lee Berry* Quadratic Chabauty for rational points on hyperelliptic curves
10:30 - 11:00	Coffee break	
11:00 - 11:30	*Teri Cowen* Ternary Lattices and Modular Forms	*Robin Ammon* Distribution of Ray Class Groups of Random Number Fields
11:30 - 12:00	*Lewis Combes* What is a Bianchi modular form? And why does anyone care?	*Ross Paterson* Elliptic Curves and Quadratic Fields

Friday 2. August		
12:00 - 12:30	*Yan Yau Cheng* Cohomological Pairings arising from Arithmetic Topological Quantum Field Theories	*William Stephenson* Raising to the power of i in a product of Elliptic Curves
12:30 - 14:30	Lunch	
14:30 - 15:00	*Edwina Aylward* Artin twists of elliptic curves and BSD	*Xiang Li* Introduction of Chabauty-Kim Methods on S-unit Equations
15:00 - 15:30	*Alexandru Pascadi* Density theorems for GL_n via Rankin-Selberg L-functions	*Zonglin Li* Geometry and distribution of roots of quadratic congruences

Plenary talks

The weight part of Serre's conjecture and theta cycles

Hanneke Wiersema

Serre's modularity conjecture states that two-dimensional mod p Galois representations arise from modular forms. For such a representation, the strong form of the conjecture specifies the weight of the corresponding modular form. This statement is called the weight part of Serre's conjecture. In this talk we introduce theta cycles and discuss the role they play in the proof of the weight part of Serre's conjecture. We then show how theta cycles can be used to answer weight questions in generalisations of the weight part of Serre's conjecture.

Langlands reciprocity and non-abelian class field theory

Andrew Wiles

I will try to outline how Langlands reciprocity may be approached in a manner similar in style to the classical approach to class field theory.

Short talks - titles & abstracts

When Rankin-Selberg periods meet Euler systems

Alexandros Groutides

Seemingly unrelated at first glance, the local representation theory of p-adic groups and Rankin-Selberg period integrals turn out to have deep connections with Euler system norm-relations. In this talk, we will briefly touch upon this idea, initially introduced by Loeffler-Skinner-Zerbes, and discuss recent developments in the integral version of the theory. Using this representation theoretic framework, we show that the local Euler factors appearing in the construction of the motivic Rankin-Selberg Euler system for $\mathrm{GL}_2 \times \mathrm{GL}_2$ are integrally optimal; i.e. any construction of this type with any choice of integral input data would give local factors appearing in tame norm relations at p which are integrally divisible by the Euler factor $\mathcal{P}'_p(\mathrm{Frob}_p^{-1})$ modulo $p-1$.

Local-global compatibility beyond the self-dual case

Bence Hevesi

In this talk, I will discuss results on local-global compatibility for the automorphic Galois representations constructed by Harris–Lan–Taylor–Thorne and Scholze some ten years ago.

Serre's Modularity conjecture

Calle Sönne

Serre's modularity conjecture is a conjecture about the modularity of "mod p" Galois representations. This can be viewed as an instance of the "mod p Langlands philosophy". On the other hand, modularity lifting theorems are statements about when one can deduce the modularity of a p-adic Galois representation from knowing that it is modular mod p. At first glance, one might think that modularity lifting theorems are in a way "orthogonal" to Serre's conjecture. This talk aims to give an idea of the suprising role that modularity lifting theorems play in the proof of Serre's conjecture.

Kloosterman sums over reductive groups

Catinca Mujdei

The classical Kloosterman sum is an exponential sum that is fundamental in the theory of automorphic forms, appearing for example in the Fourier coefficients of classical Poincaré series. The sum is usually denoted by $S(m, n; c)$, with parameters $m, n \in Z$ and $c \in \mathbb{N}$, and it ranges over certain integers modulo c. From a more general perspective, the sum can also be seen as a character sum over certain double cosets of $\mathrm{GL}_2(\mathbb{Q}_p)$. In this talk, I will explain how this perspective leads to the definition of Kloosterman sums on arbitrary reductive groups over local fields, with an explicit description in terms of their Bruhat decomposition.

Quaternionic modular forms mod p

Yiannis Fam

In a 1987 letter, Serre relates the actions of Hecke operators on modular forms mod p and functions on a certain quaternion algebra depending on p. We explain a version of this result over other quaternion algebras, working in the context of modular forms over Shimura curves.

Denominators of BSD quotients

David Kurniadi Angdinata

The Birch–Swinnerton-Dyer conjecture relates the central L-value of an elliptic curve to a quotient of algebraic invariants. This quotient is a rational number whose denominator is bounded by Mazur's torsion theorem. In this talk, I will give a tight bound on this denominator, which arises from a recent classification of 3-adic Galois images.

A few remarks about p-adic L-functions

Frederick Thøgersen

p-adic L-functions play an important role in modern number theory with key roles in the Iwasawa main conjectures and Block-Kato conjectures. We will consider the concept of p-adic L-functions using the classical modular forms case of Pollack and Stevens as a gentle introduction. With this as a basis, we will discuss more recent progress where we consider the anti-cyclotomic p-adic L-function of the definite unitary group U_{2n}. We will use this more involved case to observe the role of branching laws/ spherical varieties with respect to interpolation, and compare iwahoric and parahoric methods.

Cohomology of locally symmetric spaces

Ho Leung Fong

Starting from the work of Eichler-Shimura and Matsushima, it is realised that the cohomology of some adelic quotients of a reductive group G encodes much information about automorphic representations of G. There are many applications of this idea, such as constructions of Galois representations, rationality of special values of L-functions... In this talk, I will try to explain some of these.

Coefficients of modular polynomials (mod 23)

Jenny Roberts

At the Arithmetic Statistics spring school held in Luminy in May 2023, we were introduced to modular polynomials, both the classical example $\Phi_p(X, Y)$ and an another, $\Gamma_p(X, Y)$. A few of us took up the challenge of trying to compute the polynomial $\Gamma_p(X, Y)$ for small values of p and studying its coefficients. In doing so, we made some rather surprising discoveries about its coefficients when considered modulo 23. In this talk, I'll give an overview of what we found and the progress we've made to understanding our findings. This is joint work with Sven Cats.

Augmented Iwasawa algebras in the Langlands program

Vincenzo Di Bartolo

Let $F/\mathbb{Q}_p$ be a finite extension, O its ring of integers and $d \geq 1$ an integer. In recent formulations of a categorical perspective for the p-adic Langlands program a study of the local coherence of the category of smooth O-linear representations of $\mathrm{GL}_d(F)$ becomes relevant. We aim at showing how as well as presenting further advances in these directions.

Picard groups of affinoid algebras

Mark Heavey

Picard groups arise in numerous circumstances throughout geometry; for instance, in arithmetic geometry it is closely connected to the étale fundamental group of the given space. My research is focused on understanding the structure of smooth affinoid spaces over p-adic local fields. The general approach is to relate the Picard group of the given affinoid space with the Picard group of the reduction of a formal model. This reduction is a scheme of finite type over a finite field, which often has a Picard group whose structure as an abelian group is well understood.

Theta linkage maps and the weight part of Serre's conjecture

Martin Ortiz

The weight part of Serre's conjecture seeks to understand mod p congruences between modular eigenforms of different weights. This can be generalized to arbitrary reductive groups using the Betti cohomology of local systems on locally symmetric spaces. In the case where one has a Shimura variety available one can also try to study this problem via their coherent cohomology, though this approach has barely been explored beyond GL_2/F (with work of Edixhoven, Diamond-Sasaki etc). In that direction I will introduce the concept of theta linkage maps: some weight shifting differential operators that can produce such congruences for general Shimura varieties. Beyond GL_2 these operators mirror some of the complexity of the modular representation theory on the Betti side. I will briefly explain the classical case of $\mathrm{GL}_2/\mathbb{Q}$ and then move on to the example of $\mathrm{GSp}_4/\mathbb{Q}$ where these new phenomenon can be appreciated.

A Dirichlet series attached to modular forms of orthogonal groups

Rafail Psyroukis

A Dirichlet series involving the Fourier–Jacobi coefficients of two Siegel modular forms has been extensively studied, mainly due to its connection with the spinor L-function. In this talk, we will explain how these results can be generalised to modular forms for orthogonal groups of real signature $(2, n)$. We will start by first discussing basic facts about such modular forms and then proceed to explain the connection of the Dirichlet series of interest to the standard L-function of the orthogonal group. We will end the talk by discussing the cases for which we obtain clear-cut Euler product expressions and explain the motivation behind them.

On the unramified Fontaine-Mazur and Boston conjectures

Simon Alonso

In this talk I will present the unramified Fontaine-Mazur conjecture and its strenghtening by Boston as well as an equivalence result obtained by Calegari and Allen in the residually irreducible case. Then, I will give the first step towards a generalization of this result in the residually reducible case as well as ideas to continue in that direction.

Hilbert modular forms and geometric modularity in quadratic case

Siqi Yang

Let F be a totally real field in which p is unramified and ρ a two-dimensional mod p representation of the absolute Galois group of F that is irreducible, continuous, and totally odd. We say ρ is geometrically modular if it comes from a mod p Hilbert eigenform and algebraically modular if its Hecke eigensystem appears from the Hecke eigensystem of the spaces of mod p automorphic forms on definite quaternion algebras. It is conjectured by Diamond and Sasaki that, if k lies in a certain minimal cone, then ρ being geometrically modular of a weight (k, l) implies algebraic modularity of the same weight. In this talk, I will focus on the real quadratic case.

Ternary Lattices and Modular Forms

Teri Cowen

In the 70's Birch defined a Hecke action on isometry classes of ternary quadratic lattices of square-free discriminant and noticed that the corresponding eigenvalues are explained by weight two modular forms. In 2016, Hein fully proved this correspondence and described its precise image. Following on from this, work has been done by Hein, Tornaria and Voight towards the non-square discriminant case. In this talk, I will provide an overview of these results and will discuss ongoing work concerning the discriminant p^2 case.

What is a Bianchi modular form? And why does anyone care?

Lewis Combes

Bianchi modular forms have grown in prominence in recent years, partly due to their unique placement in the general theory of the Langlands program. They are in a "Goldilocks zone" of being not-too-hard to define, but finding themselves wrapped up all manner of unexplained phenomena. In this talk, I hope to deliver on the title by explaining what these strange creatures are, and advertising some interesting problems in their study. No knowledge of Bianchi modular forms is needed to enjoy this talk (but some familiarity with classical modular forms is recommended).

Cohomological Pairings arising from Arithmetic Topological Quantum Field Theories

Yan Yau Cheng

Inspired by the Arithmetic Topology philosophy that primes in a number ring should be analogous to knots in a 3-manifold, Arithmetic Quantum Field Theories were first developed by Minhyong Kim as a way to generate arithmetic invariants.

In this talk I will introduce the basic ideas behind Arithmetic QFTs, and pairings of cohomology classes that arise from particular ATQFTs. Time permitting, I will give examples of computations of these pairings.

Artin twists of elliptic curves and BSD

Edwina Aylward

The factorization of L-functions of elliptic curves over number fields into the product of L-functions of "Artin twists" predicts non-trivial arithmetic properties of the curves. In this talk I will describe a means of predicting positive rank for families of rational elliptic curves over finite Galois extensions. This involves using a conjectural "Artin formalism" for the BSD terms of the twisted L-functions, as well as some representation theory of finite groups.

Density theorems for GL_n via Rankin-Selberg L-functions

Alexandru Pascadi

A great number of arithmetic results depend on the best progress towards the Ramanujan-Petersson conjecture and its Archimedean counterpart, Selberg's eigenvalue conjecture. These concern the sizes of the Hecke and Laplacian eigenvalues of automorphic forms for congruence subgroups of $\mathrm{SL}_2(\mathbb{Z})$, corresponding to the local parameters of GL_2-automorphic representations. The GL_n-setting has also attracted significant interest, partly because it leads to bounds for GL_2 via symmetric power lifts.

We'll discuss very recent work (joint with Jared Lichtman) on density theorems for cuspidal automorphic representations of GL_n over $\mathbb{Q}$, which fail the generalized Ramanujan conjecture at some place. We depart from previous approaches based on Kuznetsov-type trace formulae, and instead rely on L-function techniques. This improves recent results of Blomer near the threshold of the pointwise bounds.

K3 surfaces with everywhere good reduction

Alvaro Gonzalez Hernandez

When studying elliptic curves or, more generally, abelian varieties defined over a number field, a very important source of information are the primes of bad reduction. It is therefore a natural question to ask, do abelian varieties with no primes of bad reduction exist? Turns out, the answer to this question depends on the number field over which the abelian variety is defined! Fontaine proved in 1985 that over the rationals these abelian varieties with everywhere good reduction cannot exist, but over some quadratic extensions of the rationals we can find many examples. In this talk, I will discuss what happens when instead of studying the problem of finding abelian surfaces with everywhere good reduction, one studies this same problem for K3 surfaces, a class of surfaces which can be loosely described as "the abelian surfaces weird cousins from Essex".

Continued Fractions in the Field of p-adic Numbers

Antau Yang

Continued fractions have been extensively explored over the reals. But there are still a lot of open problems for the ones over the p-adic field. A natural attempt is to find any analogue results. For many years, people have developed new algorithms and have analyzed their performance. I will quickly summarize the known results and share what I have tried so far.

Introduction to divisibility sequences for elliptic curves with complex multiplication

Edison Au-Yeung

An elliptic divisibility sequence is an integer recurrence sequence related to denominators of the integer multiples of a rational point on an elliptic curve. However, if the elliptic curve has complex multiplication, then it makes sense for us to index the sequence with the endomorphism ring instead of integers only. In this talk, we will examine existing method(s) of doing so.

Diagonal del Pezzo surfaces of degree 2 with a Brauer-Manin obstruction

Harry Shaw

Given a variety over the rationals, a natural question to ask is whether it has a rational point. If such a point exists, then the variety must have an adelic point. If the reverse implication holds, we say the variety satisfies the Hasse principle. It is well-known that the Hasse principle does not always hold, however it is conjectured by Colliot-Thélène that for rationally connected varieties the failure of the Hasse principle is explained by the Brauer-Manin obstruction. In this talk we will first discuss the Brauer-Manin obstruction (with specific focus on del Pezzo surfaces of degree 2). We will then discuss the difficulties of counting the Brauer-Manin obstruction on this family of surfaces, and the methods used to circumvent these difficulties.

Wild conductor exponents of trigonal and tetragonal curves

Harry Spencer

The problem of provably computing conductor exponents of curves is solved for curves of genus at most 3, for hyperelliptic curves away from 2 and partially in some other cases. We show how to compute the wild parts of conductor exponents of trigonal and tetragonal curves away from 2 and 3. This is done by reducing to the hyperelliptic case using isogenies between Jacobians, so in particular it won't be necessary to worry about what a conductor is!

Descent on elliptic surfaces and the Brauer-Manin obstruction

Harvey Yau

The Brauer-Manin obstruction gives a powerful way to prove the non-existence of rational points on varieties, which works when simpler methods do not. Finding examples of the Brauer-Manin obstruction requires knowledge of the Brauer group of a variety, and in this talk I will discuss how to calculate parts of the Brauer group of an elliptic surface, using methods analogous to descent on an elliptic curve.

Parity in dihedral extensions

Jamie Bell

Suppose we have an elliptic curve E with rational coefficients, and K a Galois extension of $\mathbb{Q}$ with Galois group G. Then $E(K) \otimes \mathbb{Q}$ is a rational G-representation, with dimension equal to the rank of $E(K)$. We can decompose it in to irreducible representations, and ask about their multiplicity. There is a conjecture for the parity of these, in terms of local data, which in some cases is known to follow from finiteness of sha. This allows us to predict, for example, that a curve has points over a D_{42}-extension, even when it doesn't over any Galois subfields. I will discuss, using a numerical example, how we can attempt to show this without assuming the parity conjecture.

Counting number fields whose conductor is the sum of two squares

Julie Tavernier

In this talk we will consider the number of abelian field extensions of a number field with fixed Galois group G whose conductor is the sum of two squares. The number of such extensions can be found by employing techniques from harmonic analysis and class field theory. These results can be extended to abelian field extensions whose conductor satisfies some frobenian condition, of which being the sum of two squares is an example.

Computing the Brauer-Manin obstruction on del Pezzo surfaces of degree 4

Katerina Santicola

In this talk, I'll go over some of the difficulties and lessons learned in writing Magma code that computes the BMO on del Pezzo surfaces of degree 4. Some challenges faced include linear programming, Hasse-Minkowski over number fields, and computing points on surfaces over finite fields efficiently. The spirit of this talk is to give a very gentle introduction to working with Magma, and to provide a forum for me to air my grievances.

Faltings's Theorem and Isolated Points

Kenji Terao

Faltings's theorem on rational points is a cornerstone of arithmetic geometry. In this talk, we will explore some of the machinery needed to prove Faltings's theorem, and see how this motivates the notion of isolated points. Following on from this, we will see how the study of isolated points on modular curves is linked to interesting questions on elliptic curves and Galois representations. Finally, we will survey some open questions on this topic.

Variants of the modular method

Diana Mocanu

We are going to briefly introduce the modular method for solving Diophantine equations and talk about some classical approaches. Then, we will talk about a higher dimensional variant that originated in Darmon program, where we replace Frey elliptic curves by Jacobians of "Frey hyperelliptic curves". If time permits, I will present some ongoing work involving computing conductors of these Jacobians using cluster pictures.

Isogeny relations in products of families of elliptic curves

Luca Ferrigno

We talk about "unlikely intersections" whenever we have a non-empty intersection between algebraic varieties that, for dimensional reasons, we do not expect to intersect. This expectation is the basis for many important results and conjectures in Diophantine geometry, such as Faltings' Theorem (Mordell Conjecture), Raynaud's Theorem (Manin-Mumford Conjecture), the André-Oort Conjecture (proved by Pila, Shankar and Tsimerman), and the Zilber-Pink conjecture, which is still open. In this talk, I will give an introduction to the Zilber-Pink conjecture and present a new result about the conjecture for a curve in the product of two fibered powers of elliptic schemes, provided that the curve satisfies a certain condition on the degrees of some of its coordinates.

Isogeny graphs of Abelian surfaces

Eda Kirimli

Isogeny-based cryptography is an active research area in post-quantum cryptography, and its security depends on variants of isogeny problems, namely the problem of finding an explicit isogeny between two abelian varieties. Although much of the research focused on isogenies of elliptic curves so far, it is interesting to understand isogeny graphs in dimension, and the recent attacks using isogenies in higher dimensions switched the attention to dimension two. Unlike known methods, we use very geometrical and arithmetical objects, called refined Humbert invariants. We will present applications of abelian surfaces with these invariants for isogeny-based cryptography.

Perfect powers in elliptic divisibility sequences

Maryam Nowroozi

Let E be an elliptic curve over the rationals given by an integral Weierstrass model and let P be a rational point of infinite order. The multiple nP has the form $(A_n/B_n^2, C_n/B_n^3)$ where A_n, B_n, C_n are integers with A_nC_n and B_n coprime, and B_n positive. The sequence (B_n) is called the elliptic divisibility sequence generated by P. In this talk we answer the question posed in 2007 by Everest, Reynolds and Stevens: does the sequence (B_n) contain only finitely many perfect powers?

Quadratic Chabauty for rational points on hyperelliptic curves

Lee Berry

The method of Chabauty and Coleman is a powerful tool for the explicit computation of rational points on higher genus curves. However, this method is limited to curves where the Mordell-Weil rank of the Jacobian is less than the genus of the curve. To extend its applicability, Kim introduced a significant generalisation of the classical Chabauty-Coleman method around 20 years ago. Effective applications of this theory, such as quadratic Chabauty for rational points, have been particularly useful in determining rational points on curves beyond the reach of the Chabauty-Coleman method. In this talk, we will provide a brief overview of Chabauty-Kim theory and discuss recent advancements in the application of the quadratic Chabauty method to hyperelliptic curves.

Distribution of Ray Class Groups of Random Number Fields

Robin Ammon

Despite being an important subject in number theory, for a long time the structure of the ideal class group of a number field seemed very mysterious. Taking a new approach to understand it, H. Cohen and H. Lenstra started to study the statistical behaviour of class groups and realised that this behaviour appears to be governed by a fundamental principle about the distribution of random mathematical objects. They made influential conjectures about the distribution of ideal class groups which have sparked a lot of research in arithmetic statistics and beyond, and recently lead people to wonder also about the distribution of close relatives of ideal class groups, the ray class groups.

In my talk, I will give an introduction to Cohen and Lenstra's conjectures and discuss the ideas of arithmetic statistics and the fundamental principle that underlie them. I will then talk about work in progress that aims to generalise the conjectures to ray class groups, and which also has implications to other statistical questions.

Elliptic Curves and Quadratic Fields

Ross Paterson

Given an elliptic curve $E/\mathbb{Q}$ and a quadratic field K, we will be interested in the group of K-rational points $E(K)$. It possesses two natural subgroups: $E(\mathbb{Q})$, and $E'(\mathbb{Q})$, where the latter is the set of rational points on the associated twist of E. It is well known that the ranks of $E(\mathbb{Q})$ and $E'(\mathbb{Q})$ sum to the rank of $E(K)$, but do they actually generate it? We'll discuss what is known in this direction!

Raising to the power of i in a product of Elliptic Curves

William Stephenson

In this talk I will explain some ongoing research of my own in which I prove an analogue of a result of Jonathan Pila. He proved that varieties inside certain products of the multiplicative group, $\mathbb{G}_m$, have a finite number of "plu-optimal" subvarieties. I aim to outline a proof of this in the case where the ambient variety is a product of elliptic curves.

Introduction of Chabauty-Kim Methods on S-unit Equations

Xiang Li

The Chabauty-Kim method is a p-adic method to find rational or integral points on algebraic curves effectively. Kim conjectured that his machinery will give the solution sets eventually. In the talk, I will first introduce the Chabauty-Kim methods, and some known results on verifying Kim's conjecture on S-unit equations where S is a set of primes of size 2. Later, I will discuss about the case when S is of size 3 (which is not known) to give an upper bound of the number of solutions using Chabauty-Kim methods and weight filtrations, and also discuss about the Chabuaty-Kim methods over number fields if time permitted.

Geometry and distribution of roots of quadratic congruences

Zonglin Li

The recent paper by Marklof and Welsh established limit laws for the fine-scale distribution of the roots of the quadratic congruence $\mu^2 \equiv D (\operatorname{mod} m)$, ordered by the modulus m, where D is a square-free positive integer and $D \not\equiv 1 (\operatorname{mod} 4)$. This is achieved by relating the roots (when $D > 0$) to the tops of certain geodesics in the Poincare upper half-plane and (when $D < 0$) to orbits of points, under the action of the modular group.

In this talk, we will investigate the remaining case when $D > 1$ is a square-free integer and $D \equiv 1 (\operatorname{mod} 4)$. We will understand how the roots can be related to the tops of the geodesics by considering ideals of the ring $\mathbb{Z}\left[\sqrt{D}\right]$ and the ring of integers of the quadratic number field $\mathbb{Q}\left(\sqrt{D}\right)$. This is joint work with Matthew Welsh.
